CHAPTER 1

Real and Financial Capital

A firm is essentially a transformation function (held together as a "nexus of contracts") that takes certain inputs and transforms them into outputs.[1] In this transformation process, *capital* has two altogether distinct conceptual meanings. The first is the traditional notion of capital as a factor of production—some kind of asset that helps a firm transform inputs into widgets. In order to finance the required investments in such capital and to disperse the risks of the firm's assets among a pool of investors, firms issue securities. These securities are *also* referred to as capital.

The two concepts are, of course, related—*too* related, unfortunately, despite representing different sides of the traditional corporate balance sheet. It is precisely this close connection that can lead to a fairly significant amount of confusion, especially when it comes to discussing the relation between risk and capital in a practical capital management exercise such as capital budgeting. A banker, for example, is quite likely to define capital budgeting as the allocation of capital to business units. The role played by risk in that exercise is in computing some risk-adjusted return on capital at risk that serves as the hurdle rate or performance measure for these attributions of risk capital to business risks. A corporate treasurer faced with a capital budgeting problem, by contrast, will more likely faithfully compute the net present values of all projects under consideration in order to decide which ones to pursue as new investments in hard assets. Risk comes into the picture through the weighted average cost of capital—itself a risk-adjusted measure of expected returns—used to discount the future risky cash flows on the investment project.

Neither perspective is wrong per se. In fact, we'll see in Chapter 5 the conditions under which the two approaches imply *the same decision rule* for whether to accept a new investment project. Clearly, though, the potential for confusion is enormous. Our sole objective in this introductory chapter is to eliminate those sources of confusion by introducing the concepts of capital in a careful, systematic way—specifically, by considering

the relationships of the market values of both real capital and financial capital to the market value of the firm. Not surprisingly, we will conclude that they are equal!

In the process of showing this result, we also accomplish two other objectives. First, we provide a quick review of how corporate securities can be viewed through an "options contract lens." Second, we develop the notion of an economic balance sheet for a firm—a concept that will be very simplistic in this chapter, but which will play an important role and become increasingly complex as we move forward through Part One and the rest of the book.

REAL CAPITAL AND THE VALUE OF THE FIRM

A *real asset* is any asset that can be consumed or used directly.[2] The ability of people or organizations to consume or use a real asset is what gives that asset its value.[3] More colloquially, we sometimes say that real assets have value because we can either eat them, give them to someone else to eat, or use them to produce something edible.

A *real capital asset*—or just *real capital*—is a specific type of real asset that contributes to the production of a sequence of goods or services over time. What distinguishes a real capital asset from any other real asset is mainly the time dimension. Real capital is involved in medium- and long-term production, whereas real *non*capital assets are often consumed immediately. A large dump truck is a real capital asset, for example, whereas an apple is a real *non*capital asset intended more for immediate consumption. Other examples of real capital include plants, equipment, patented production processes and technologies, and the like.

A *firm* is a collection of real assets held together through a nexus of contracts between various parties, including laborers and capitalists, contractors, customers, and the like. What gives the firm its value is the cash flow that the assets owned by and entrusted to the firm may produce over time. Myers (1977) usefully suggests that the market value of the real assets of the firm—denoted $A(t)$ at any time t—be divided into two components:[4]

$$A(t) = V^A(t) + V^G(t) \tag{1.1}$$

where $V^A(t)$ = time t market value of the firm's assets in place
$V^G(t)$ = time t market value of the firm's growth opportunities

Assets in Place

A firm's assets in place at time t are those real capital assets that the firm has already bought and paid for with prior investments. The current market value of assets already in place is equal to the discounted present value (PV) of the future net cash flows on those assets plus the firm's current-period net cash flow from operations, or

$$V^A(t) = \overbrace{\sum_{j=1}^{\infty} \frac{E[X(t+j)]}{1+E[R^A(t,\ t+j)]}}^{\text{Discounted Expected PV of Future Net Cash Flows}} + \overbrace{[X(t)-I(t)]}^{\text{Current Net Operating Cash Flow}}$$
$$= \overbrace{\sum_{j=0}^{\infty} \frac{E[X(t+j)]}{1+E[R^A(t,\ t+j)]} - I(t)}^{\text{Market Value of Assets in Place}} \tag{1.2}$$

where $X(t + j)$ = time $t + j$ net cash flows on current assets

The future expected net cash flows are discounted at the rate of capitalization appropriate to the risk of the stream of future net cash flows—that is, at the expected return on current assets over the relevant time period, $E[R^A(t, t + j)]$.

In equation (1.2), net cash flows X are aggregated across all parts of the firm. We include in these quantities, moreover, any noncapital or other operating income and expenditures so that the value of current assets in place is an exhaustive representation of all the firm's current and future cash flows excluding only the cash flows from growth opportunities (to be discussed shortly). We will retain this assumption through this book unless we explicitly state otherwise.

We can also view the firm as a portfolio or bundle of specific projects or business units. If the firm has U such units and the cash flows from these projects are all mutually exclusive and exhaustive relative to the representation in equation (1.2), then we can rewrite the current market value of the firm's assets in place as the sum of the market values of the projects or operating divisions of the firm, any of which is denoted $V_j^A(t)$ at time t:

$$V^A(t) = \sum_{u=1}^{U} V_u^A(t) = \sum_{u=1}^{U} \sum_{j=0}^{\infty} \frac{E[X_u(t+j)]}{1+E[R^A(t,\ t+j)]} - I_u(t) \tag{1.3}$$

where $X_u(t + j)$ = time $t + j$ net cash flows on current assets in business unit u
$I_u(t)$ = time t investment expenditure on assets in business unit u

Growth Opportunities

Myers (1977) defines growth opportunities as future opportunities the firm will have to acquire or develop an asset. If the firm decides to make a subsequent investment expenditure, the current growth opportunity will become a future asset in place. Otherwise, the growth opportunity will expire worthless. Growth opportunities may be either strategic decisions the firm faces that have value in their own right or actual future investment opportunities that can be identified today.

As an example, suppose the firm in question is a pharmaceutical company. At some future date $t + j$, the firm has the opportunity to begin developing a new drug that will be designated "Project Q." The firm will incur a series of known future investment expenditures to develop the drug over time, but we treat this sequence of expenditures as a single expenditure made at time $t + j$ equal to the discounted present value of all future investment expenditures for the drug's development. Denote this investment expenditure as $I_Q(t + j)$ at time $t + j$. If the firm undertakes the project, its net present value will be

$$\overbrace{\sum_{k=1}^{\infty} \frac{E[X_Q(t+j+k)]}{1+E[R^A(t+j,\ t+j+k)]}}^{\text{Discounted Expected PV of Future Cash Flows}} - \overbrace{I_Q(t+j)}^{\text{PV of All Investment Expenditures}} \tag{1.4}$$

where future cash flows come from drug sales revenues, patent licensing revenues, and so on. Alternatively, the firm may decide not to incur the investment expense and to forgo the project. The value at time $t + j$ of Project Q to develop the drug thus is

$$\max\left\{\sum_{k=1}^{\infty} \frac{E[X_Q(t+j+k)]}{1+E[R^A(t+j,\ t+j+k)]} - I_Q(t+j),\ 0\right\} \tag{1.5}$$

Growth opportunities are more commonly known as *real options* because of their call option–like features that are apparent in equation (1.5). Specifically, any particular growth opportunity can be viewed as a call option on the value of a future asset in place with a strike price equal to the investment expenditure required to develop or acquire that asset. We generally associate real options with strategic asset acquisitions or with latent and intangible assets, such as intellectual property.

The market value of a growth opportunity at any time t is just the time t price of the real option that can be exercised at time $t + j$. The market value of all K growth opportunities the firm has identified is just the sum of all the real option prices corresponding to each of the K future projects, decisions, or opportunities:

$$V^G(t) = \sum_{k=1}^{K} V_k^G(t) \tag{1.6}$$

Value of the Firm

The market value of a firm is equal to the market value of its real capital, which, in turn, is just the sum of the market values of the firm's assets in place and growth opportunities:

$$V(t) = A(t) = V^A(t) + V^G(t)$$
$$= \overbrace{\sum_{u=1}^{U}\sum_{j=0}^{\infty} \frac{E[X_u(t+j)]}{1+E[R^A(t,\ t+j)]} - I_u(t)}^{\text{Market Value of Assets in Place in U Operating Divisions}} + \overbrace{\sum_{k=1}^{K} V_k^G(t)}^{\text{Market Value of K Growth Opportunities}} \tag{1.7}$$

FINANCIAL CAPITAL AND THE VALUE OF THE FIRM

A *financial asset* is just a claim on the cash flows generated by one or more real assets. Financial assets come in numerous forms and are created for a variety of reasons, all of which are intended in some way to assist individuals in the consumption, production, and/or exchange of real assets or to assist corporations in some aspect of their business activities.

When a financial asset is issued by a corporation, we call that asset *financial capital* for the corporate issuer. Corporations generally issue financial capital for several reasons. The first and most obvious function of financial capital is raising funds to help the issuing firm finance its investments by exchanging a claim on the future cash flows of the firm's

real assets for current cash. In addition, issuing financial capital pools investments in the firm's assets to diversify the risks of those assets across multiple investors. Financial capital, of course, also carries the same benefits of other financial assets, such as facilitating a change of control of a bundle of assets (e.g., a firm) without forcing the real assets to be exchanged.

As was the case with real capital as compared to real assets, the main distinction between financial capital and financial assets is generally the perceived longer-term nature of the latter. In addition, financial capital involves *some* degree of risk in the business enterprise for investors in such claims. In that manner, the firm both finances its operations *and* spreads the risk of its investments across multiple investors whenever it issues financial capital.

Basic Forms of Financial Capital

Securities issued as financial capital by a corporation represent claims on the future net cash flows of that business enterprise and come in essentially two different forms: *equity* and *debt*. We will discuss a wide range of hybrids that fall somewhere between the textbook definitions of equity and debt, but we save that discussion for later (Chapter 14).

Equity and debt capital are distinguished along principally two dimensions. The first is governance. For the most part, holders of a firm's common stock and certain types of preference shares are the only claim holders who can take an active role in the firm's management and operations.[5] The second distinction concerns the nature of the claim itself, which in turn affects the risk/reward profile of the two types of securities.

Equity is known as a *residual claim* because it gives holder a claim on the net cash flows of a firm remaining *after* the firm has paid all of its fixed claims and bills. For publicly listed and traded firms, the most basic form of equity claim is common stock, which is essentially a proportional interest in the firm's residual net cash flows. Common stock is generally a "perpetual" security—it has no stated date on which the corporate issuer buys back the security for a defined price. Equity also comes in the forms of preferred stock or preference shares, shares in a general or limited partnership, and the like. Equity holders of a firm can earn income from their claims by reselling them or waiting for the firm to close its doors and pay a liquidating dividend. In addition to these two ways of turning shares into a capital gain (or loss), some firms also choose to compensate their equity claim holders with periodic cash distributions or dividends.

The second type of financial capital that a firm can issue to raise cash—called *debt*—is a loan in which the firm promises to repay principal

and/or interest according to some predetermined schedule.[6] Unlike equity, the maximum payoff on debt is stated explicitly in the debt contract, so that debt holders do not receive a higher payoff when the firm is more profitable. In return for less of an upside, debt holders also bear less business risk than equity holders because they are paid off before equity holders. That does not mean that debt bears no business risk. As the risk of a default on debt rises, the expected return on the debt will rise accordingly to induce creditors to continue holding the debt. The closer to default, the more the debt begins to look like equity even despite having a specified maximum payoff.

Unlike common equity shares, debt generally does have a stated maturity date. Accordingly, debt holders usually earn income from their claims according to the schedule of any interim interest payments promised over the life of the debt and the principal repayment promised at maturity. Debt may also contain early redemption options for either issuers (i.e., callable debt) or investors (i.e., puttable debt), in which principal is payable on the early redemption date. In addition, like equity, debt holders can also earn income prior to redemption by selling their claims to other investors or if the firm becomes insolvent and enters liquidation.

Interest paid to holders of debt securities is similar in spirit to dividends paid to equity holders. As a practical matter, however, there are two key differences. First, unlike dividends, interest on debt is defined in advance for the whole term of the debt contract.[7] Second, because debt holders have priority over equity holders in receiving cash distributions, interest on debt is payable *before* any dividends can be disbursed to equity holders.

Seniority, Priority, and Subordination

In the event that the value of the firm's assets is below the promised payments on the fixed claims the firm has issued, we know that equity holders receive nothing and debt holders receive a pro rata distribution of the remaining assets. Most corporations, however, have a richer variety of securities than just a single class each of debt and common stock.

The concept of *priority* refers to the preference given to certain claim holders when the firm becomes insolvent and the proceeds from the liquidation of a firm's assets must be distributed. The *seniority* of a claim refers to the priority of claim holders. Alternatively, the *depth of subordination* of a security is the inverse of its seniority—that is, the most deeply subordinated securities have the lowest seniority and the lowest priority in the event of insolvency. In general, the higher the depth of subordination of a given class of security, the greater the default risk of the security for a given market value of assets. The reason is that claim holders with a given priority cannot

receive any payments in the event the firm becomes insolvent until all claim holders senior to them are paid off first.

Exhibit 1.1 illustrates the related concepts of priority, subordination, and seniority in financial capital. The exhibit depicts the economic balance sheet of a firm at time *t* that owns assets that back financial capital in the form of debt and equity. The financial capital of the firm is shown on the right side of the economic balance sheet in the order of increasing seniority/priority (i.e., decreasing depth of subordination/increasing risk) from bottom to top. Although we continue to ignore any corporate securities apart from plain-vanilla debt and equity, we now allow for multiple classes and depths of subordination within each category.

The lowest-priority claimant on a corporation is always the holder of some type of equity or residual interest in the firm. In fact, equity claims are sometimes called "soft" claims because their seniority in the capital structure of the firm adjusts to the other securities that the firm issues so as to remain the most junior claim outstanding. The most junior and softest claim is usually common stock, the class of equity that receives nothing in the event of insolvency unless all other security holders have first been made whole.

Senior to common stock but junior to all else is preferred stock or preference shares. Preferred stock is another type of equity, but with features more closely resembling debt such as a stated maturity date and often limited voting rights. Dividends on common stock cannot be paid until dividends on preferred stock have been paid. But at the same time, preference shares are equity and not debt, thus implying that the preferred dividend is discretionary. Whereas the failure of a corporation to make a promised interest payment on debt is an event of default, the preferred stock dividend can usually be suspended without triggering default provisions.[8]

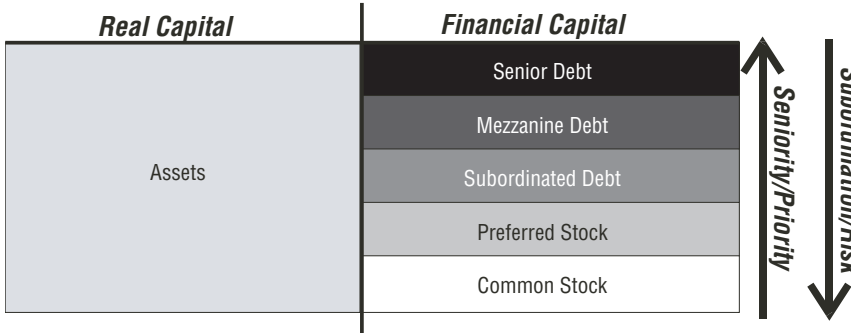

EXHIBIT 1.1 Seniority/Priority/Subordination of Financial Capital

Debt as a type of financial capital often has several levels of subordination, with the lowest priority going to subordinated or junior debt and the highest to senior creditors. An unsecured senior creditor to a firm is the holder of an unsecured debt claim that is the most senior unsecured security in the firm's capital structure. (We will deal with secured debt in Part Three of the book.) Senior debt often comes in the form of bank loans made directly by commercial banks to corporations.

Subordinated debt is debt in which the lenders receive a pro rata share of the cash proceeds from liquidated assets after senior lenders have been paid off. It is possible to have multiple levels of subordinated debt, often held by different types of firms. Banks, for example, may hold a firm's senior subordinated (senior sub) debt, and public or institutional investors may hold the same firm's junior subordinated debt.

In terms of credit risk and external credit ratings, the most junior tranche of subordinated debt is generally two rating levels below the senior tranche. Although this makes junior sub debt relatively much riskier than senior debt, the *absolute* risk of the debt depends on the overall risk of the firm. For a start-up or a highly leveraged and thinly capitalized firm, subordinated debt may well be "junk."[9] But for a well-capitalized AAA-rated issuer, junior sub debt might well still carry a AA+ or AA rating.

Capital Structure

The capital structure of a corporation is, very simply, the relative mixture of fixed and residual claims that a firm issues. We can examine a firm's mixture of fixed and residual claims in terms of either stocks or flows.

If we adopt a stock perspective of capital structure, we think in terms of ratios of outstanding debt to equity or to total financial capital *at any given point in time*. The capital structure of a firm is essentially the mixture of debt and equity issued by a firm, together with the subordination and maturities of those claims.

The most common way of describing a firm's capital structure if we adopt a stock perspective in lieu of a flow perspective is the *leverage ratio*, or the percentage of the firm's total financial capital that is in the form of fixed rather than residual claims. A leverage ratio of .30 or 30 percent, for example, means that 30 percent of the capital structure of the firm is comprised of fixed income obligations, with the remaining 70 percent in the form of equity claims. We can evaluate these ratios, moreover, in terms of either book or market values depending on what we are trying to accomplish.

Alternatively, the firm's capital structure can also be described in terms of flows. If we think of capital structure in terms of *flows of funds*, we will

tend to focus on variables like dividend payout ratios and interest service coverage ratios. Or we can think of capital structure in terms of *flows of securities issued*, which indicates a firm's preference for issuing different types of claims to manage risk or finance new investments.

Capital Structure and Organizational Form

Fama and Jensen (1983a, 1983b, 1985) define four types of organizations, distinguished principally by the nature of the financial capital claims they issue and by the governance model for the firm—more specifically, the relationship between who holds the financial capital claims issued by the firm and who governs the firm.

First, an *open corporation* issues residual claims usually in the form of unrestricted common stock. Common stock entitles each shareholder to a proportional claim on the net cash flows or value of the assets of the firm. If the firm has issued N shares, each shareholder will receive $S(T)/N$ upon liquidation of the firm on date T or a similar proportional dividend payment in any preliquidation period that the firm pays dividends.

Shares issued by open corporations can be freely bought and sold in a secondary market once they have been issued. Although the owner of a share is recorded at the securities registrar for the issuing company, the investor alone can decide when to buy or sell it. The company's permission is usually not required for a transfer of share ownership to occur.[10]

Second, a *closed corporation* or *proprietorship* also issues residual claims in the form of equity shares, but the equity of a closed corporation usually *cannot* be bought and sold freely. In proprietorships, equity shares usually take the form of partnership shares or interests. These interests are often obtainable only by managers of the firm and cannot usually be sold or transferred to just anyone, unlike common stock which *can* be freely bought and sold by about anyone with the cash to buy it. In limited partnerships, equity shares may not be conditional on management responsibilities, but are still usually bought and sold under highly restrictive conditions that tend to limit the number of potential partners to a prespecified group of investors with whom the company wants to deal.

The third type of Fama/Jensen organization is a *financial mutual* or *syndicate*. The residual claimants in these types of firms are also the customers of the firms. Shares in an open-end mutual fund or real estate investment trust (REIT), for example, represent pro rata claims on the assets in which the fund or trust invests the proceeds it receives from share sales. But the only reason the fund/trust has collected funds from investors in the first place is to reinvest these funds on behalf of investors in some specific

asset class or investment program. This may seem like circular logic, when in fact it is merely evidence that the residual claimants of the firm are also its *users*.

Shares in some financial mutuals are listed for trading in organized markets, whereas others are available only through private negotiations or auctions. A share in a country club, for example, is a share in a mutual in which the share purchasers also use the facilities of the club. In this case, the purchaser likely must obtain the membership share directly from the club and its governing members, must meet certain membership criteria, and may not necessarily sell her membership to the average man on the street without permission of the other governing members.

Finally, *nonprofits* are organizations that have no residual claimants per se. The closest thing are the donors and supporters who provide operating cash flows directly. Instead of receiving a residual claim on the net cash flows of the nonprofit, donors receive an intangible residual claim on the fruits of the nonprofit's labors.

The choice of organizational form has a strong relationship to the firm's capital structure. Remember that issuing financial capital is a way for a firm not only to raise money, but also to pool investment in the firm so that the business risk of the firm is dispersed among a group of investors. The risk-bearing attributes of securities issued by different firms often relate to the organizational form chosen. Because an open corporation, for example, has achieved a separation of ownership and control, equity is intended mainly as fund-raising device. Because it also allows investors to share in the risks and rewards of the business, it will be priced accordingly. But because equity ownership is separated from direct managerial control, the risks of equity do not play an incentive role. On the contrary, these risks can give rise to conflicts between security holders and management when the latter does not act in the best interests of the former.

A small partnership, by contrast, is often characterized by more consolidation of ownership and control. Equity in a partnership thus serves not just as a means of raising funds, but also to give the partners of the firm an ongoing incentive to invest their own time and effort in keeping the firm as profitable as possible.

Value of the Firm . . . Again

The combined value of all the firm's securities at any time t—denoted $W(t)$—is just the sum of the market values of the debt and equity claims issued by the firm, regardless of the maturity structure and subordination of

those claims. The total market value of the firm's financial capital at time t thus is

$$W(t) = [S(t) + \delta(t)] + [D(t) + \rho(t)] \tag{1.8}$$

where $S(t)$ = time t market value of the firm's equity or share capital outstanding at the end of time $t - 1$
$\delta(t)$ = dividends paid at time t to shareholders
$D(t)$ = time t market value of the firm's debt outstanding at the end of time $t - 1$
$\rho(t)$ = interest paid at time t to creditors

Provided that the aggregate value of all financial claims exhaustively represents all the firm's cash flows arising from its assets in place and growth opportunities, the combined wealth of the securities issued by the firm must equal the combined value of the firm's assets. Combining equations (1.1) and (1.8) thus leads us to conclude that

$$V(t) = A(t) = W(t) \tag{1.9}$$

In other words, *the market value of the firm is always equal to the market value of the real assets held by the firm, which in turn is also equal to the market value of the aggregate financial capital issued by the firm.*

Note that equation (1.9) is a tautology and it *always* holds, provided *only* that the financial claims issued by a firm exhaustively represent all the cash flows on the assets owned by the firm. To see why this is indeed the case, we can utilize an options-theoretic perspective. This also provides a good chance early in the book to review the most rudimentary option structures at a level sufficient for even readers with limited background to digest the references to options later in the book.

FINANCIAL CAPITAL THROUGH AN OPTIONS LENS

Financial claims derive their value solely from the cash flows of the portfolio of real assets held by the firm issuing those claims. As long as the firm issues at least one residual class of security to absorb fluctuations in the value of its real assets, the sum of the market prices of all financial capital claims will always exactly equal the mark-to-market value of the firm's real capital. We can use options theory to verify this is indeed the case—and to provide us with a quick review of options theory at a level that should be more than adequate to digest later references to options in this book.

For concreteness in what follows, consider a firm that issues common stock and zero coupon debt that matures on date T with a total face value of FV. Suppose the debt is divided into senior and subordinated classes with face values of X and Y, respectively, so that $FV = X + Y$. For simplicity, assume that $A(T)$ indicates the market value of the firm's net assets on date T, where we define net assets as the firm's real assets less any current liabilities due at T other than debt. Further assume the firm liquidates its real assets on date T at the market price of $A(T)$ for cash and costlessly redistributes that cash to investors in its three classes of securities.

Debt

Senior debt is a fixed claim on the cash flows of the firm that pays X to investors as long as the firm's net assets are sufficient to cover that outlay. If $A(T) < X$, senior creditors to the firm receive as a group a pro rata distribution of whatever cash was obtained from liquidating the firm's net assets for $A(T)$. The payoff to senior debt when it matures thus can be written logically as

$$D^{sr}(T) = X - \max[X - A(T), 0] \tag{1.10}$$

Viewed through an option product lens, senior debt thus can be viewed as the combination of a riskless loan of X plus a *short* put on the firm's assets struck at X. The payoff to senior debt holders is shown graphically as a function of $A(T)$ in Exhibit 1.2.

The interpretation of senior debt is fairly intuitive. If the firm is doing well and $A(T)$ is relatively high, bondholders receive their fixed commitment X in full but never participate in the firm's profitability with payoffs

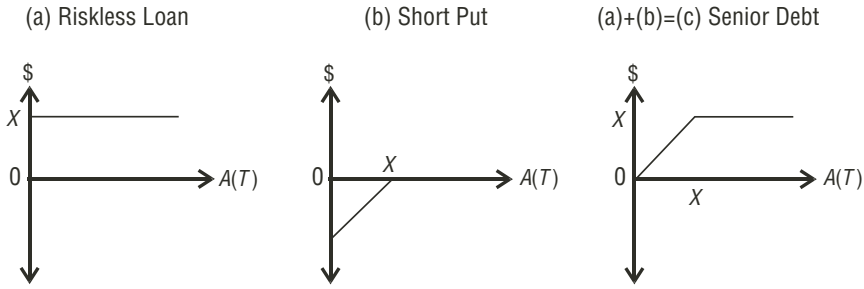

EXHIBIT 1.2 Value of Zero Coupon Senior Debt at Maturity

above that amount. But if the firm's assets fall below X so that the firm is in default, senior creditors as a group will receive whatever is left.

Subordinated debt, in turn, is a fixed claim on the cash flows of the firm that pays Y to investors as long as the firm's net assets are sufficient to cover that outlay. But because subordinated debt is junior in capital structure to senior debt, the junior creditors receive nothing until senior debt has been completely retired. Full repayment of the subordinated debt thus requires that the firm have assets worth at least $X + Y$ to cover both payments.

By analogy to senior debt, we can view the *gross* payoff to subordinated debt as a riskless loan of $X + Y$ plus a short put on the firm's assets struck at $X + Y$. The *net* payoff, of course, must also take into account that part of that gross total has been set aside for senior creditors. So, we must subtract the value of the senior debt from the gross payoff on the subordinated debt to get the net payoff to junior creditors:

$$\begin{aligned} D^{sub}(T) &= (X + Y) - \max[(X + Y) - A(T), 0] \\ &\quad - \{X - \max[X - A(T), 0]\} \\ \rightarrow D^{sub}(T) &= Y - \max[(X + Y) - A(T), 0] + \max[X - A(T), 0] \end{aligned} \tag{1.11}$$

The payoff to subordinated debt in (1.11) is shown graphically in Exhibit 1.3, which readers will recognize as the payoff on a long or bullish vertical spread consisting of a riskless loan Y, a long put struck at X, and a short put struck at $X + Y$. As before, the payoff is again intuitive if you look at the graph. For asset levels below X, subordinated debt receives no payment at all because all the firm's assets have gone to pay off senior debt. For asset values above X but below $X + Y$, junior creditors get a pro rata portion of their total claim on the firm. And for assets with values above $X + Y$, junior debt is fully repaid and reaches its maximum cash inflow of Y.

Comparing the payoffs of senior and subordinated debt in equations (1.10) and (1.11) and in Exhibits 1.2 and 1.3, we can see that the put written by senior debt holders has essentially been purchased by junior creditors. This is the essence of subordination and seniority. The long put essentially puts a "wall" between subordinated debt and senior debt that cannot be crossed in the event that the firm is insolvent, thus guaranteeing that subordinated debt holders cannot try and make a claim on assets of the firm when assets are below the face value of senior debt.

Further reinforcing this concept, we can see that the wall separating junior and senior creditors—the put struck at X—disappears when we aggregate the total value of all outstanding debt issued by the firm:

$$D(T) = D^{sr}(T) + D^{sub}(T) = FV - \max[FV - A(T), 0] \tag{1.12}$$

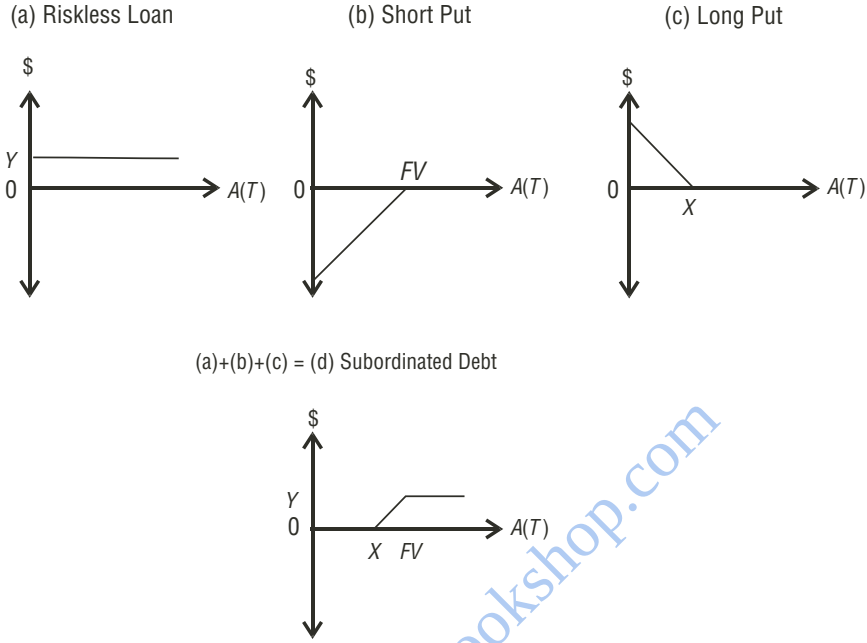

EXHIBIT 1.3 Value of Zero Coupon Subordinated Debt at Maturity

So, debt *as a whole* has the payoff of a cash loan of $X + Y$ plus a short put struck at $X + Y$. The long put struck at X helps us determine how junior and senior creditors split this total among themselves, but it does not affect the total.

Equity

Common stock is a residual claim on the net cash flows of the firm. Common stockholders thus get a payoff at T equal to gross proceeds of the liquidation of the firm's assets minus all the bills the firm has yet to pay, including repaying senior and junior creditors. Common stock is a limited liability contract, so in the event that the firm's assets are not sufficient to cover a full repayment of debt, equity is simply worthless. The payoff to common stockholders as a group thus is

$$S(T) = \max[A(T) - FV, 0] \tag{1.13}$$

So, equity can be viewed as a long call with a strike price equal to the book value of all outstanding debt, $FV = X + Y$. For any net asset values $A(T)$ that are above the principal due on the two classes of bonds, equity holders as a group gain dollar for dollar. This is the "residual" on which equity holders have a claim, and its potential upside is limited only by the potential increase in the value of the firm's assets. In return for this potential reward, equity is last in line from a subordination standpoint and thus has no value unless the residual net asset value is positive after debt is repaid. The payoff in equation (1.13) on equity as a group is shown graphically in Exhibit 1.4.

Conceptually, we can think about the value of equity from both gross and net standpoints, just as we did with subordinated debt. On a gross basis, equity gets the value of the remaining assets of the firm $A(T)$ when the firm goes into liquidation on date T. But payments on all claims senior to equity must be subtracted from this total to get the net payoff to equity. So, equity has a payoff of

$$S(T) = A(T) - \{FV - \max[FV - A(T), 0]\} \tag{1.14}$$

In other words, equity holders get all the cash proceeds from the en masse liquidation of the firm's assets but then must immediately spend $X + Y = FV$ to repay all debt holders. The expression in equation (1.14) is equivalent to the payoff on a call option we wrote in equation (1.13).[11]

The Value of the Firm

Just as our put option struck at X acted as a wall between junior and senior debt to preserve seniority, the put option struck at FV in the payoffs for debt and equity shown in equations (1.12) and (1.14) now acts as a similar wall to separate equity holders as a class from debt holders as a

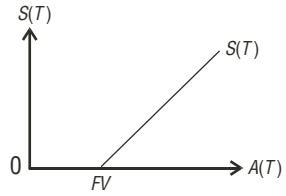

EXHIBIT 1.4 Value of Equity on Debt Maturity Date

class for subordination purposes. Recall from before that this wall went away when we added the values of sub and senior debt to get the total market value of debt. Similarly, if we now add the values of total debt and equity to get the total value of all outstanding financial capital issued by the firm, the wall again vanishes, leaving us with

$$W(T) = A(T)$$

As was the case within the debt class of securities, the same thing is true for the firm as a whole—how we apportion payments between security holders does not affect the total that we have to apportion.[12] Exhibit 1.5 shows the result graphically.

All of our examples have focused on the date that debt matures. If we look at any other date, all the relations explored still hold, although the graphs are not as pretty. Nevertheless, the above results hold for all time periods.

We have also been ignoring dividends and interest on debt. But if we now add those back, we can confirm our earlier assertion in equation (1.9) that the value of the firm is equal to the value of all financial capital issued by the firm *and* the value of the firm's real assets:

$$W(t) = [S(t) + \delta(t)] + [D(t) + \rho(t)] = A(t) = V(t) \qquad (1.15)$$

As we said earlier—and now we can see why—equations (1.8) and (1.15) are a tautology. Absolutely the only assumption we need to make for these equations to hold true is that the firm's financial capital must include at least one residual claim. If the firm issues only debt, then you can see the residual value of the firm's remaining assets will never balance out. But with at least one equity claim, the equity security always absorbs whatever residual market value is left over after debt holders are repaid.

ECONOMIC BALANCE SHEET OF THE FIRM

With these concepts in hand, we can now write down the economic balance sheet of the firm, sometimes called the mark-to-market balance sheet of the firm. We must emphasize that this is not an accounting concept. In particular, items that we would consider "off balance sheet" under accounting rules will show up on the firm's economic balance sheet.

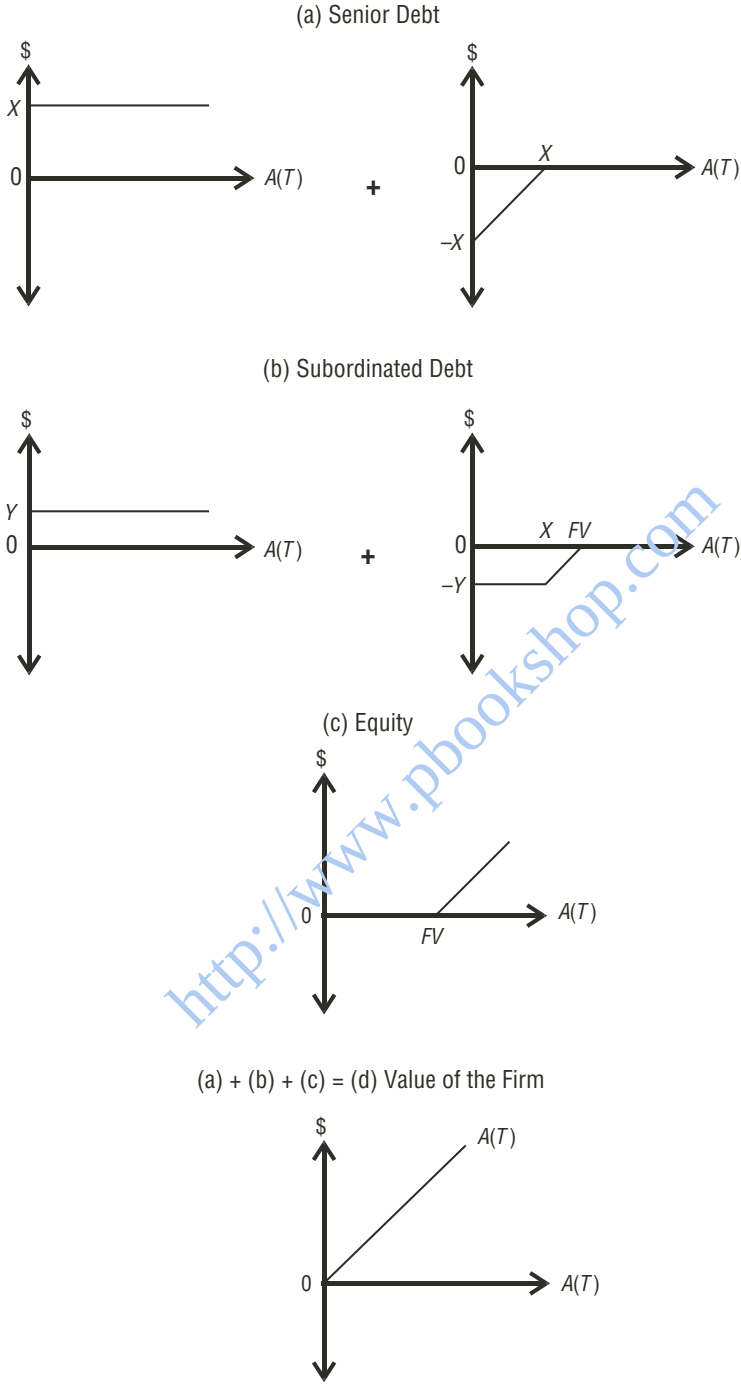

EXHIBIT 1.5 Value of the Firm on Debt Maturity Date

Assets	Liabilities and Equity
$A(t)$	$D(t)$ $S(t)$

EXHIBIT 1.6 Economic Balance Sheet of the Firm

Our uses of certain terms like "financial capital," moreover, are economic. Accountants would define financial capital as shareholders' equity plus depreciation and retained earnings, whereas the economic definition of financial capital is the sum of the market values of all the firm's debt and equity securities.

The economic balance sheet for a firm that issues a single class each of debt and equity is shown in Exhibit 1.6. This shows the fundamental relationship between real and financial capital—namely, that the market values of the two must be equal and that this is the market value of the firm. This economic balance may seem a simple construct now, but we will make extensive use of it later when things are not so simple.