

# Contents

<b>List of Figures</b>	<b>xiii</b>
<b>List of Tables</b>	<b>xvii</b>
<b>List of Examples</b>	<b>xx</b>
<b>Foreword</b>	<b>xxii</b>
<b>Preface to Volume II</b>	<b>xxvi</b>
<b>II.1 Factor Models</b>	<b>1</b>
II.1.1 Introduction	1
II.1.2 Single Factor Models	2
II.1.2.1 Single Index Model	2
II.1.2.2 Estimating Portfolio Characteristics using OLS	4
II.1.2.3 Estimating Portfolio Risk using EWMA	6
II.1.2.4 Relationship between Beta, Correlation and Relative Volatility	8
II.1.2.5 Risk Decomposition in a Single Factor Model	10
II.1.3 Multi-Factor Models	11
II.1.3.1 Multi-factor Models of Asset or Portfolio Returns	11
II.1.3.2 Style Attribution Analysis	13
II.1.3.3 General Formulation of Multi-factor Model	16
II.1.3.4 Multi-factor Models of International Portfolios	18
II.1.4 Case Study: Estimation of Fundamental Factor Models	21
II.1.4.1 Estimating Systematic Risk for a Portfolio of US Stocks	22
II.1.4.2 Multicollinearity: A Problem with Fundamental Factor Models	23
II.1.4.3 Estimating Fundamental Factor Models by Orthogonal Regression	25
II.1.5 Analysis of Barra Model	27
II.1.5.1 Risk Indices, Descriptors and Fundamental Betas	28
II.1.5.2 Model Specification and Risk Decomposition	30
II.1.6 Tracking Error and Active Risk	31
II.1.6.1 Ex Post versus Ex Ante Measurement of Risk and Return	32
II.1.6.2 Definition of Active Returns	32
II.1.6.3 Definition of Active Weights	33
II.1.6.4 Ex Post Tracking Error	33

II.1.6.5	Ex Post Mean-Adjusted Tracking Error	36
II.1.6.6	Ex Ante Tracking Error	39
II.1.6.7	Ex Ante Mean-Adjusted Tracking Error	40
II.1.6.8	Clarification of the Definition of Active Risk	42
II.1.7	Summary and Conclusions	44
<b>II.2</b>	<b>Principal Component Analysis</b>	<b>47</b>
II.2.1	Introduction	47
II.2.2	Review of Principal Component Analysis	48
II.2.2.1	Definition of Principal Components	49
II.2.2.2	Principal Component Representation	49
II.2.2.3	Frequently Asked Questions	50
II.2.3	Case Study: PCA of UK Government Yield Curves	53
II.2.3.1	Properties of UK Interest Rates	53
II.2.3.2	Volatility and Correlation of UK Spot Rates	55
II.2.3.3	PCA on UK Spot Rates Correlation Matrix	56
II.2.3.4	Principal Component Representation	58
II.2.3.5	PCA on UK Short Spot Rates Covariance Matrix	60
II.2.4	Term Structure Factor Models	61
II.2.4.1	Interest Rate Sensitive Portfolios	62
II.2.4.2	Factor Models for Currency Forward Positions	66
II.2.4.3	Factor Models for Commodity Futures Portfolios	70
II.2.4.4	Application to Portfolio Immunization	71
II.2.4.5	Application to Asset–Liability Management	72
II.2.4.6	Application to Portfolio Risk Measurement	73
II.2.4.7	Multiple Curve Factor Models	76
II.2.5	Equity PCA Factor Models	80
II.2.5.1	Model Structure	80
II.2.5.2	Specific Risks and Dimension Reduction	81
II.2.5.3	Case Study: PCA Factor Model for DJIA Portfolios	82
II.2.6	Summary and Conclusions	86
<b>II.3</b>	<b>Classical Models of Volatility and Correlation</b>	<b>89</b>
II.3.1	Introduction	89
II.3.2	Variance and Volatility	90
II.3.2.1	Volatility and the Square-Root-of-Time Rule	90
II.3.2.2	Constant Volatility Assumption	92
II.3.2.3	Volatility when Returns are Autocorrelated	92
II.3.2.4	Remarks about Volatility	93
II.3.3	Covariance and Correlation	94
II.3.3.1	Definition of Covariance and Correlation	94
II.3.3.2	Correlation Pitfalls	95
II.3.3.3	Covariance Matrices	96
II.3.3.4	Scaling Covariance Matrices	97
II.3.4	Equally Weighted Averages	98
II.3.4.1	Unconditional Variance and Volatility	99
II.3.4.2	Unconditional Covariance and Correlation	102
II.3.4.3	Forecasting with Equally Weighted Averages	103

II.3.5	Precision of Equally Weighted Estimates	104
II.3.5.1	Confidence Intervals for Variance and Volatility	104
II.3.5.2	Standard Error of Variance Estimator	106
II.3.5.3	Standard Error of Volatility Estimator	107
II.3.5.4	Standard Error of Correlation Estimator	109
II.3.6	Case Study: Volatility and Correlation of US Treasuries	109
II.3.6.1	Choosing the Data	110
II.3.6.2	Our Data	111
II.3.6.3	Effect of Sample Period	112
II.3.6.4	How to Calculate Changes in Interest Rates	113
II.3.7	Equally Weighted Moving Averages	115
II.3.7.1	Effect of Volatility Clusters	115
II.3.7.2	Pitfalls of the Equally Weighted Moving Average Method	117
II.3.7.3	Three Ways to Forecast Long Term Volatility	118
II.3.8	Exponentially Weighted Moving Averages	120
II.3.8.1	Statistical Methodology	120
II.3.8.2	Interpretation of Lambda	121
II.3.8.3	Properties of EWMA Estimators	122
II.3.8.4	Forecasting with EWMA	123
II.3.8.5	Standard Errors for EWMA Forecasts	124
II.3.8.6	RiskMetrics™ Methodology	126
II.3.8.7	Orthogonal EWMA versus RiskMetrics EWMA	128
II.3.9	Summary and Conclusions	129
<b>II.4</b>	<b>Introduction to GARCH Models</b>	<b>131</b>
II.4.1	Introduction	131
II.4.2	The Symmetric Normal GARCH Model	135
II.4.2.1	Model Specification	135
II.4.2.2	Parameter Estimation	137
II.4.2.3	Volatility Estimates	141
II.4.2.4	GARCH Volatility Forecasts	142
II.4.2.5	Imposing Long Term Volatility	144
II.4.2.6	Comparison of GARCH and EWMA Volatility Models	147
II.4.3	Asymmetric GARCH Models	147
II.4.3.1	A-GARCH	148
II.4.3.2	GJR-GARCH	150
II.4.3.3	Exponential GARCH	151
II.4.3.4	Analytic E-GARCH Volatility Term Structure Forecasts	154
II.4.3.5	Volatility Feedback	156
II.4.4	Non-Normal GARCH Models	157
II.4.4.1	Student t GARCH Models	157
II.4.4.2	Case Study: Comparison of GARCH Models for the FTSE 100	159
II.4.4.3	Normal Mixture GARCH Models	161
II.4.4.4	Markov Switching GARCH	163
II.4.5	GARCH Covariance Matrices	164
II.4.5.1	Estimation of Multivariate GARCH Models	165
II.4.5.2	Constant and Dynamic Conditional Correlation GARCH	166
II.4.5.3	Factor GARCH	169

II.4.6	Orthogonal GARCH	171
II.4.6.1	Model Specification	171
II.4.6.2	Case Study: A Comparison of RiskMetrics and O-GARCH	173
II.4.6.3	Splicing Methods for Constructing Large Covariance Matrices	179
II.4.7	Monte Carlo Simulation with GARCH Models	180
II.4.7.1	Simulation with Volatility Clustering	180
II.4.7.2	Simulation with Volatility Clustering Regimes	183
II.4.7.3	Simulation with Correlation Clustering	185
II.4.8	Applications of GARCH Models	188
II.4.8.1	Option Pricing with GARCH Diffusions	188
II.4.8.2	Pricing Path-Dependent European Options	189
II.4.8.3	Value-at-Risk Measurement	192
II.4.8.4	Estimation of Time Varying Sensitivities	193
II.4.8.5	Portfolio Optimization	195
II.4.9	Summary and Conclusions	197
<b>II.5</b>	<b>Time Series Models and Cointegration</b>	<b>201</b>
II.5.1	Introduction	201
II.5.2	Stationary Processes	202
II.5.2.1	Time Series Models	203
II.5.2.2	Inversion and the Lag Operator	206
II.5.2.3	Response to Shocks	206
II.5.2.4	Estimation	208
II.5.2.5	Prediction	210
II.5.2.6	Multivariate Models for Stationary Processes	211
II.5.3	Stochastic Trends	212
II.5.3.1	Random Walks and Efficient Markets	212
II.5.3.2	Integrated Processes and Stochastic Trends	213
II.5.3.3	Deterministic Trends	214
II.5.3.4	Unit Root Tests	215
II.5.3.5	Unit Roots in Asset Prices	218
II.5.3.6	Unit Roots in Interest Rates, Credit Spreads and Implied Volatility	220
II.5.3.7	Reconciliation of Time Series and Continuous Time Models	223
II.5.3.8	Unit Roots in Commodity Prices	224
II.5.4	Long Term Equilibrium	225
II.5.4.1	Cointegration and Correlation Compared	225
II.5.4.2	Common Stochastic Trends	227
II.5.4.3	Formal Definition of Cointegration	228
II.5.4.4	Evidence of Cointegration in Financial Markets	229
II.5.4.5	Estimation and Testing in Cointegrated Systems	231
II.5.4.6	Application to Benchmark Tracking	239
II.5.4.7	Case Study: Cointegration Index Tracking in the Dow Jones Index	240
II.5.5	Modelling Short Term Dynamics	243
II.5.5.1	Error Correction Models	243

II.5.5.2	Granger Causality	246
II.5.5.3	Case Study: Pairs Trading Volatility Index Futures	247
II.5.6	Summary and Conclusions	250
<b>II.6</b>	<b>Introduction to Copulas</b>	<b>253</b>
II.6.1	Introduction	253
II.6.2	Concordance Metrics	255
II.6.2.1	Concordance	255
II.6.2.2	Rank Correlations	256
II.6.3	Copulas and Associated Theoretical Concepts	258
II.6.3.1	Simulation of a Single Random Variable	258
II.6.3.2	Definition of a Copula	259
II.6.3.3	Conditional Copula Distributions and their Quantile Curves	263
II.6.3.4	Tail Dependence	264
II.6.3.5	Bounds for Dependence	265
II.6.4	Examples of Copulas	266
II.6.4.1	Normal or Gaussian Copulas	266
II.6.4.2	Student $t$ Copulas	268
II.6.4.3	Normal Mixture Copulas	269
II.6.4.4	Archimedean Copulas	271
II.6.5	Conditional Copula Distributions and Quantile Curves	273
II.6.5.1	Normal or Gaussian Copulas	273
II.6.5.2	Student $t$ Copulas	274
II.6.5.3	Normal Mixture Copulas	275
II.6.5.4	Archimedean Copulas	275
II.6.5.5	Examples	276
II.6.6	Calibrating Copulas	279
II.6.6.1	Correspondence between Copulas and Rank Correlations	280
II.6.6.2	Maximum Likelihood Estimation	281
II.6.6.3	How to Choose the Best Copula	283
II.6.7	Simulation with Copulas	285
II.6.7.1	Using Conditional Copulas for Simulation	285
II.6.7.2	Simulation from Elliptical Copulas	286
II.6.7.3	Simulation with Normal and Student $t$ Copulas	287
II.6.7.4	Simulation from Archimedean Copulas	290
II.6.8	Market Risk Applications	290
II.6.8.1	Value-at-Risk Estimation	291
II.6.8.2	Aggregation and Portfolio Diversification	292
II.6.8.3	Using Copulas for Portfolio Optimization	295
II.6.9	Summary and Conclusions	298
<b>II.7</b>	<b>Advanced Econometric Models</b>	<b>301</b>
II.7.1	Introduction	301
II.7.2	Quantile Regression	303
II.7.2.1	Review of Standard Regression	304
II.7.2.2	What is Quantile Regression?	305
II.7.2.3	Parameter Estimation in Quantile Regression	305

II.7.2.4	Inference in Linear Quantile Regression	307
II.7.2.5	Using Copulas for Non-linear Quantile Regression	307
II.7.3	Case Studies on Quantile Regression	309
II.7.3.1	Case Study 1: Quantile Regression of Vftse on FTSE 100	
Index		309
II.7.3.2	Case Study 2: Hedging with Copula Quantile Regression	314
II.7.4	Other Non-Linear Regression Models	319
II.7.4.1	Non-linear Least Squares	319
II.7.4.2	Discrete Choice Models	321
II.7.5	Markov Switching Models	325
II.7.5.1	Testing for Structural Breaks	325
II.7.5.2	Model Specification	327
II.7.5.3	Financial Applications and Software	329
II.7.6	Modelling Ultra High Frequency Data	330
II.7.6.1	Data Sources and Filtering	330
II.7.6.2	Modelling the Time between Trades	332
II.7.6.3	Forecasting Volatility	334
II.7.7	Summary and Conclusions	337
<b>II.8</b>	<b>Forecasting and Model Evaluation</b>	<b>341</b>
II.8.1	Introduction	341
II.8.2	Returns Models	342
II.8.2.1	Goodness of Fit	343
II.8.2.2	Forecasting	347
II.8.2.3	Simulating Critical Values for Test Statistics	348
II.8.2.4	Specification Tests for Regime Switching Models	350
II.8.3	Volatility Models	350
II.8.3.1	Goodness of Fit of GARCH Models	351
II.8.3.2	Forecasting with GARCH Volatility Models	352
II.8.3.3	Moving Average Models	354
II.8.4	Forecasting the Tails of a Distribution	356
II.8.4.1	Confidence Intervals for Quantiles	356
II.8.4.2	Coverage Tests	357
II.8.4.3	Application of Coverage Tests to GARCH Models	360
II.8.4.4	Forecasting Conditional Correlations	361
II.8.5	Operational Evaluation	363
II.8.5.1	General Backtesting Algorithm	363
II.8.5.2	Alpha Models	365
II.8.5.3	Portfolio Optimization	366
II.8.5.4	Hedging with Futures	366
II.8.5.5	Value-at-Risk Measurement	367
II.8.5.6	Trading Implied Volatility	370
II.8.5.7	Trading Realized Volatility	372
II.8.5.8	Pricing and Hedging Options	373
II.8.6	Summary and Conclusions	375
	<b>References</b>	<b>377</b>
	<b>Index</b>	<b>387</b>