

Contents

Preface	xv
About the Authors	xix
CHAPTER 1	
Introduction	1
1.1 The Need for Better Financial Modeling of Asset Prices	1
1.2 The Family of Stable Distribution and Its Properties	5
1.2.1 Parameterization of the Stable Distribution	5
1.2.2 Desirable Properties of the Stable Distributions	7
1.2.3 Considerations in the Use of the Stable Distribution	8
1.3 Option Pricing with Volatility Clustering	9
1.3.1 Non-Gaussian GARCH Models	11
1.4 Model Dependencies	12
1.5 Monte Carlo	13
1.6 Organization of the Book	14
References	15
CHAPTER 2	
Probability Distributions	19
2.1 Basic Concepts	19
2.2 Discrete Probability Distributions	20
2.2.1 Bernoulli Distribution	21
2.2.2 Binomial Distribution	21
2.2.3 Poisson Distribution	22
2.3 Continuous Probability Distributions	22
2.3.1 Probability Distribution Function, Probability Density Function, and Cumulative Distribution Function	23
2.3.2 Normal Distribution	26
2.3.3 Exponential Distribution	28
2.3.4 Gamma Distribution	28

2.3.5	Variance Gamma Distribution	29
2.3.6	Inverse Gaussian Distribution	30
2.4	Statistic Moments and Quantiles	30
2.4.1	Location	31
2.4.2	Dispersion	31
2.4.3	Asymmetry	31
2.4.4	Concentration in Tails	32
2.4.5	Statistical Moments	32
2.4.6	Quantiles	34
2.4.7	Sample Moments	35
2.5	Characteristic Function	35
2.6	Joint Probability Distributions	39
2.6.1	Conditional Probability	39
2.6.2	Joint Probability Distribution Defined	40
2.6.3	Marginal Distribution	41
2.6.4	Dependence of Random Variables	41
2.6.5	Covariance and Correlation	42
2.6.6	Multivariate Normal Distribution	43
2.6.7	Elliptical Distributions	46
2.6.8	Copula Functions	47
2.7	Summary	54
	References	54
CHAPTER 3		
	Stable and Tempered Stable Distributions	57
3.1	α -Stable Distribution	58
3.1.1	Definition of an α -Stable Random Variable	58
3.1.2	Useful Properties of an α -Stable Random Variable	61
3.1.3	Smoothly Truncated Stable Distribution	63
3.2	Tempered Stable Distributions	65
3.2.1	Classical Tempered Stable Distribution	65
3.2.2	Generalized Classical Tempered Stable Distribution	68
3.2.3	Modified Tempered Stable Distribution	69
3.2.4	Normal Tempered Stable Distribution	70
3.2.5	Kim-Rachev Tempered Stable Distribution	73
3.2.6	Rapidly Decreasing Tempered Stable Distribution	75
3.3	Infinitely Divisible Distributions	76
3.3.1	Exponential Moments	80
3.4	Summary	82

<i>Contents</i>	ix
3.5 Appendix	82
3.5.1 The Hypergeometric Function	83
3.5.2 The Confluent Hypergeometric Function	83
References	84
CHAPTER 4	
Stochastic Processes in Continuous Time	87
4.1 Some Preliminaries	88
4.2 Poisson Process	88
4.2.1 Compounded Poisson Process	89
4.3 Pure Jump Process	89
4.3.1 Gamma Process	92
4.3.2 Inverse Gaussian Process	92
4.3.3 Variance Gamma Process	92
4.3.4 α -Stable Process	93
4.3.5 Tempered Stable Process	94
4.4 Brownian Motion	95
4.4.1 Arithmetic Brownian Motion	99
4.4.2 Geometric Brownian Motion	99
4.5 Time-Changed Brownian Motion	100
4.5.1 Variance Gamma Process	101
4.5.2 Normal Inverse Gaussian Process	102
4.5.3 Normal Tempered Stable Process	103
4.6 Lévy Process	104
4.7 Summary	105
References	106
CHAPTER 5	
Conditional Expectation and Change of Measure	107
5.1 Events, σ -Fields, and Filtration	107
5.2 Conditional Expectation	109
5.3 Change of Measures	111
5.3.1 Equivalent Probability Measure	111
5.3.2 Change of Measure for Continuous-Time Processes	113
5.3.3 Change of Measure in Tempered Stable Processes	117
5.4 Summary	121
References	121
CHAPTER 6	
Exponential Lévy Models	123
6.1 Exponential Lévy Models	123

x	CONTENTS
6.2	Fitting α -Stable and Tempered Stable Distributions 126
6.2.1	Fitting the Characteristic Function 126
6.2.2	Maximum Likelihood Estimation with Numerical Approximation of the Density Function 127
6.2.3	Assessing the Goodness of Fit 127
6.3	Illustration: Parameter Estimation for Tempered Stable Distributions 131
6.4	Summary 135
6.5	Appendix: Numerical Approximation of Probability Density and Cumulative Distribution Functions 135
6.5.1	Numerical Method for the Fourier Transform 139
	References 140
CHAPTER 7	
	Option Pricing in Exponential Lévy Models 141
7.1	Option Contract 141
7.2	Boundary Conditions for the Price of an Option 142
7.3	No-Arbitrage Pricing and Equivalent Martingale Measure 145
7.4	Option Pricing under the Black-Scholes Model 148
7.5	European Option Pricing under Exponential Tempered Stable Models 149
7.5.1	Illustration: Implied Volatility 152
7.5.2	Illustration: Calibrating Risk-Neutral Parameters 153
7.5.3	Illustration: Calibrating Market Parameters and Risk-Neutral Parameters Together 161
7.6	Subordinated Stock Price Model 164
7.6.1	Stochastic Volatility Lévy Process Model 166
7.7	Summary 167
	References 167
CHAPTER 8	
	Simulation 169
8.1	Random Number Generators 170
8.1.1	Uniform Distributions 170
8.1.2	Discrete Distributions 172
8.1.3	Continuous Nonuniform Distributions 172
8.1.4	Simulation of Particular Distributions 177
8.2	Simulation Techniques for Lévy Processes 182
8.2.1	Taking Care of Small Jumps 183
8.2.2	Series Representation: A General Framework 186
8.2.3	Rosiński Rejection Method 191
8.2.4	α -Stable Processes 192

<i>Contents</i>	xi
8.3 Tempered Stable Processes	193
8.3.1 Kim-Rachev Tempered Stable Case	196
8.3.2 Classical Tempered Stable Case	198
8.4 Tempered Infinitely Divisible Processes	199
8.4.1 Rapidly Decreasing Tempered Stable Case	201
8.4.2 Modified Tempered Stable Case	202
8.5 Time-Changed Brownian Motion	203
8.5.1 Classical Tempered Stable Processes	205
8.5.2 Variance Gamma and Skewed Variance Gamma Processes	206
8.5.3 Normal Tempered Stable Processes	207
8.5.4 Normal Inverse Gaussian Processes	208
8.6 Monte Carlo Methods	209
8.6.1 Variance Reduction Techniques	210
8.6.2 A Nonparametric Monte Carlo Method	214
8.6.3 A Monte Carlo Example	216
Appendix	217
References	220
CHAPTER 9	
Multi-Tail t-Distribution	225
9.1 Introduction	225
9.2 Principal Component Analysis	227
9.2.1 Principal Component Tail Functions	228
9.2.2 Density of a Multi-Tail t Random Variable	231
9.3 Estimating Parameters	232
9.3.1 Estimation of the Dispersion Matrix	233
9.3.2 Estimation of the Parameter Set Θ	233
9.4 Empirical Results	237
9.4.1 Comparison to Other Models	237
9.4.2 Two-Dimensional Analysis	238
9.4.3 Multi-Tail t Model Check for the DAX	242
9.5 Summary	244
References	246
CHAPTER 10	
Non-Gaussian Portfolio Allocation	247
10.1 Introduction	247
10.2 Multifactor Linear Model	248
10.3 Modeling Dependencies	251
10.4 Average Value-at-Risk	253
10.5 Optimal Portfolios	255

xii	CONTENTS
10.6	The Algorithm 257
10.7	An Empirical Test 259
10.8	Summary 268
	References 269
CHAPTER 11	
	Normal GARCH models 271
11.1	Introduction 271
11.2	GARCH Dynamics with Normal Innovation 272
11.3	Market Estimation 275
11.4	Risk-Neutral Estimation 278
	11.4.1 Out-of-Sample Performance 282
11.5	Summary 285
	References 285
CHAPTER 12	
	Smoothly Truncated Stable GARCH Models 287
12.1	Introduction 287
12.2	A Generalized NGARCH Option Pricing Model 288
12.3	Empirical Analysis 291
	12.3.1 Results under the Objective Probability Measure 292
	12.3.2 Explaining S&P 500 Option Prices 296
12.4	Summary 306
	References 307
CHAPTER 13	
	Infinitely Divisible GARCH Models 309
13.1	Stock Price Dynamic 311
13.2	Risk-Neutral Dynamic 312
13.3	Non-Normal Infinitely Divisible GARCH 315
	13.3.1 Classical Tempered Stable Model 315
	13.3.2 Generalized Tempered Stable Model 317
	13.3.3 Kim-Rachev Model 319
	13.3.4 Rapidly Decreasing Tempered Stable Model 322
	13.3.5 Inverse Gaussian Model 324
	13.3.6 Skewed Variance Gamma Model 326
	13.3.7 Normal Inverse Gaussian Model 329
13.4	Simulate Infinitely Divisible GARCH 331
	Appendix 332
	References 334

CHAPTER 14

Option Pricing with Monte Carlo Methods	337
14.1 Introduction	337
14.2 Data Set	338
14.2.1 Market Estimation	339
14.3 Performance of Option Pricing Models	346
14.3.1 In-Sample	346
14.3.2 Out-of-Sample	352
14.4 Summary	355
References	356

CHAPTER 15

American Option Pricing with Monte Carlo Methods	357
15.1 American Option Pricing in Discrete Time	358
15.2 The Least Squares Monte Carlo Method	359
15.3 LSM Method in GARCH Option Pricing Model	364
15.4 Empirical Illustration	365
15.5 Summary	372
References	372

Index

373

<http://www.pbookshop.com>

<http://www.pbookshop.com>