FX Options and Smile Risk

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Preface

When I first proposed writing a book on FX options, I could not help thinking that the final result would produce in the reader that disappointing, yet typically human, feeling caused by the recognition of what the Qoelet expresses in such a condensed way: "Quod factum est, ipsum est, quod faciendum est: nihil sub sole novum", which in slightly more modern words, and in accordance with the situation, means "Many books on options have been written in the past and this one is just telling the same old stories everybody knows". This fear was also sharpened by the fact that some very good books have already been written on the subject, so that just trying to be at the same level would be a titanic task. In this respect, I would like to mention here the excellent book by Uwe Wystup [03], which covers many areas, from pricing to regulation issues.

My scepticism about the likely outcome of my efforts was then partially reduced when, by chance, I read an aphorism of that solitary Colombian thinker (still inexplicably not too much known), Nicolas Davila, in his *Escolios a un texto implicito*, which stated: "Nobody thinks seriously until he cares about being original". I started to become aware that actually I did not have to search for new areas to analyse, and that I did not necessarily have to be original about the choice of subjects: "simply", I had to explore them deeply. Two questions naturally arose in my mind: Do I have the knowledge and expertise to undertake such a thorough inquiry? Besides, and probably more importantly, even if we assume that knowledge and expretise just for the sake of argument, why should I do it?

As far as the first question is concerned, I could not conceitedly say that my expertise derived from theoretical studies or technical skills, or from the fact that I was a smart trader capable of understanding the markets on any occasion, simply because none of that was true. Yet, in the year 2000, when I was working as a market maker on the interest derivatives (caps, floors and swaptions) market in Banca IMI, Milan, I was asked by the two heads of the dealing room to start a desk, market making in FX options. I had no experience in such a market, and nobody who could teach me about it worked, or had ever worked, in the bank. So I began setting up pricing systems and risk management tools by relying only on my intuition and reasoning. Then, I started to make prices and manage the book, and so started to learn. I learnt in the only way living beings learn on earth, that is: by suffering. In the market-making context suffering means basically two things: losing money in its phenomenal aspect (which mainly concerns the financial institution) and feeling depressed in its psychological aspect (which mainly concerns the trader). Ultimately, I can say I achieved my expertise on FX options by suffering, so that I have no fear in claiming that my knowledge and understanding of FX options is not purely

academic or theoretical, in which case I should admit my manifest inferiority to many people. Alternatively said, my knowledge is entirely due to the principle that the eighteenth-century philosopher Vico stated in his *Principi di Scienza Nuova*, according to which one really and fully knows something only if he has made it.

As far as the second question is concerned, it is relevant that in the year 2006 I stopped being in charge of the FX options desk in Banca IMI. I can safely say that (to use the scholastic philosopher's categories) if I was, in a more or less unconscious way, the efficient cause of the FX options desk, I was also, again in a more or less unconscious way, the final cause of it (at least in the way I liked it to operate). After two years I had stopped the market-making activity in FX options, but I did not want to forget and lose for ever all that I had assimilated during those six years. Writing a book is likely the best way to firmly fix all the concepts and the know-how that I absorbed from my experience.

As should be clear from all that has been said above, this book is written from a market-maker perspective and is focused mainly on problems related to pricing and risk management. I prefer to start with a list of what this book is not meant to be: it is not a mathematical finance textbook, although some basic options pricing theory will be presented and in general much mathematical formalism will be used; it is not aimed at showing all the possible structures that can be traded in the FX market, especially with a bank's customers (corporates, speculators, investors, etc.). Hence, I do not deal with aspects referring to the sell side. As a consequence of the previous point, I will not analyse all the possible existing kinds of contracts. Namely, I will not deal with Asian options, basket options and correlation contracts (range mountain options, for example). These options are typically used to build structured products for investors and they are very common in the equity options market. When currencies are considered as an asset class, then the same kind of options can have them as an underlying. Anyway, many books have been written on how to price such contracts, and how to manage their risk and, although they have their main reference to equities, their result can easily be extended to the FX market. In few words, this book is not a collection of pricing formulae. Besides, I will not enter into details of the interest rates market and I will not examine how to build a discount factor curve by bootstrap procedures: I assume that we are already provided with discount factors for any maturity, even if I am aware that I am neglecting a very momentous subject, at least at the time of writing.

This book is aimed at examining all the relevant issues a market maker has to cope with, both in terms of pricing different kinds of contracts and managing their related risks. Many details, often overlooked in most textbooks or articles, will be examined explicitly. Actually, they represent the link between the theory and practice, and they have a dramatic impact on the profitability of an FX options desk. I will also provide many examples: since in most cases one must resort to numerical procedures, they will be described step-by-step and then worked out in practice.

After this preliminary warning, an overview of the outline of the book is in order. I will start, in the first chapter, with the basic definitions of the FX market: the definition of pairs and the description of the main contracts are presented. I will also illustrate the main conventions operating amongst professional market makers. The second chapter is devoted to a quick review of the main concepts of the option pricing theory and their application within a Black–Scholes (BS hereon) economy, and then a stochastic volatility environment. I introduce some models that could be implemented to price and manage FX options, although in subsequent chapters I will use only one of them as an example of the alternatives to the BS setting.

Managing the volatility risk is the main task of the options trader, so the entire third chapter is devoted to the effects of volatility on the profits and losses arising from the hedging activity. It is in this regard that the volatility smile is first introduced and examined. The fourth chapter extends the analysis to the building of a consistent volatility smile from a few options' market prices. Here I take the chance to remember that much of the work related to these topics has been conducted together with Fabio Mercurio, an exceptional colleague from the quantitative department in Banca IMI, and a good friend of mine too: it was a great intellectual pleasure to work with him and I thank him for sharing with me his experience and skills.

The fifth chapter dwells on the pricing of plain vanilla options and digital options, with much attention paid to some details and market conventions whose impact on the pricing is significant. In the sixth chapter barrier options are examined; they probably from the vast majority of the exotic options dealing in the FX market, so that they deserve an in-depth analysis and many tools and methods devised by practitioners will be described. By the same token, in the seventh chapter the other less common exotic options are examined.

The eighth chapter illustrates the tools for monitoring the main risks of an FX options book; besides, it shows and comments at some length on the behaviour, in terms of volatility risks, of the plain vanilla hedging instruments and of the main exotic options. The ninth and final chapter offers a quick analysis of the links among three currencies and sketches an extension of the methods examined in the previous chapters to the contracts depending on many pairs.

One noteworthy feature of most of the methods and approaches described is that they hinge mainly on the BS model, which is still the main working tool in the market, although its flaws have been identified and discussed abundantly during the last 30 years. The reason for the striking inconsistency between the ascertained deficiencies of the BS model, and its widespread use in the FX market, is not due to the fact that market makers are stupidly stubborn (or, at least, they are not completely stupidly stubborn): on the contrary they are aware of the risks that the model is not able to consider and include them in the pricing by resorting to sophisticated, yet definitely empirical (mis-)uses of the model, sometimes designed in a very clever way, even if from a theoretical perspective the adopted solutions may make academicians turn their noses up. I would like to define this as a "Dionysian" approach to the problems related to FX options: the complexity and even the inconsistency of the real world is accepted and faced with all the means we have at our disposal, although a reasonable rigour is needed in the choice of them. In contrast, I would see an "Apollonian" approach as aimed at the perfection of the formal theory, at the elegance of the derivation of the results and the beauty of the internal consistency of the models: the fascination for all of these is manifestly congenital to human nature (at least the most noble part of it) but, alas, they are not enough to account for all the noxious details of the real world. As usually happens, a combination of the two approaches, an "Apollonian" vision of a "Dionysian" experience, as someone wrote somewhere, is likely to produce the best results. I believe this is what actually occurs in the FX options market (and in other markets too, to be honest). On the other hand, if they say that options trading is an art, then FX options trading is the *Oedipus Rex*, or the Sistine Chapel if you prefer visual works.

I do not mean to start from the origin of the universe to thank all the people and events that made possible the writing of this book, but I cannot help mentioning my parents, who wanted me to study at LUISS University in Rome; there I took a degree in Financial Markets' Economics, under the supervision of Professor Emilio Barone, with a thesis on the pricing of American options. Professor Barone, whose bright mind I admire, was the first to encourage my studies in finance and I was honoured to write with him two articles. I would like to thank all the people who worked with me on the FX options desk in Banca IMI, even if for a short

time: Roberto Binello, Marek Fogiel, Giuseppe Levato, Michele Lanza (who succeeded me as the head of the desk and who contributed greatly to itse development), Andrej Mariani, Cristina Castagner and Alessandro Gavazzeni. I would also like to mention my colleagues and friends from the interest rate options desk: Luca Dominici, Stefano Denuccio, Pierluigi D'Orazio and Davide Moresco. In the same bank I had the lucky chance to work in a stimulating environment with an exceptional quantitative department: besides the already mentioned Fabio Mercurio, I had interesting discussions with Francesco Rapisarda, Andrea Bugin, Damiano Brigo, Giulio Sartorelli and Lorenzo Bisesti. I have to acknowledge also the illuminating talks that I had with my colleagues and friends Francesco Fede, Raffaele Giura and Sergio Grasso.

Paola Mosconi deserves special thanks for proofreading the manuscript and for suggesting many improvements. The suggestions of anonymous reviewers are greatly acknowledged as well.

Although not directly related to the ideas and concepts discussed in this book, still all my friends in Milan (many of whom I have known since I was at the university) had a more or less hidden role: I would like to thank them for all their support and affection.

Finally, I must thank the last two top managers I had as my bosses in Banca IMI: Andrea Crovetto and Gianluca Cugno, whose decisions, unconsciously and unwittingly according to the utmost perfect heterogenesis of ends, ultimately allowed me to write this book.

Notation and Acronyms

- S_t : spot price of the exchange rate at time t
- $F(t, T)$: forward price of the exchange rate at time t for a contract expiring at time T
- $r^d(t)$,r^d_t : domestic spot rate at time t. It may be continuous, simple or annual compounded according to the context
- $r^f(t)$,r^f_t : foreign spot rate at time t. It may be continuous, simple or annual compounded according to the context
- $P^d(t, T) = E^Q[e^{-\int_t^T r^d(s)ds}]$: domestic zero-coupon bond price expiring at time T prevailing at time t
- $P^f(t, T) = E^Q[e^{-\int_t^T r^f(s)ds}]$: foreign zero-coupon bond price expiring at time T prevailing at time t
- $D^d(t) = D^d_t = e^{\int_0^t r^d(s)ds}$: domestic deposit (bank account) accruing interest at the domestic rate r^d with initial value in domestic currency units $D^d(0) = 1$
- $D^f(t) = D^f_t = e^{\int_0^t r^f(s)ds}$: foreign deposit (bank account) accruing interest at the foreign rate r^f with initial value in foreign currency units $D^f(0) = 1$
- H_t : barrier level at time t
- τ : time between t and T expressed as a year fraction, i.e. $\tau = \frac{T-t}{365}$
- $T_1, T_2, ..., T_i - 1, T_i$: set of maturities
- ς_t : instantaneous volatility of exchange rate spot process at time t
- $\sigma(K, T)$, $\sigma(K)$: implied volatility to plug into the Bl formula for an option struck at K and expiring in T
- Q : risk-neutral measure
- Q^T : forward risk-adjusted measure (the domestic zero-coupon $P(t, T)$ is the numeraire)
- $E[x]$: expected value of x under the physical measure
- $E^Q[x]$: expected value of x under the risk-neutral measure
- $E^T[x]$: expected value of x under the forward risk-adjusted measure
- $\mathcal{N}(\mu, \sigma)$: normal distribution with mean μ and variance σ
- $\Phi(x)$: cumulative distribution function of a standard Gaussian distribution calculated in x
- W_t, Z_t : brownian motions under the real-world measure
- W^Q_t, Z^Q_t : brownian motions under the risk-neutral measure
- $\mathcal{O}(\cdot)$: price of a European contingent claim, such as a plain vanilla European option
- Bl$(S_t, t, T, K, P^d(t, T), P^f(t, T), \sigma, \omega)$: price of a plain vanilla European option at time t and expiring at time T, struck at K and evaluated according to the BS model with a forward price of the exchange rate $F(t; T)$, an implied volatility equal to σ and with the price of the

domestic zero-coupon bond equal to $P^d(t, T)$. If the option is a call then $\omega = 1$, if it is a put then $\omega = -1$

- **C**$(\cdot)$: price of a plain vanilla European call option. The function's arguments vary according to the context
- **P**$(\cdot)$: price of a plain vanilla European put. The function's arguments vary according to the context
- p : an option's premium
- $\mathcal{E}(\cdot)$: price of a generic exotic option
- $\mathcal{B}(\cdot)$: price of a generic European barrier option, such as an up&out call option
- $\mathcal{DB}(\cdot)$: price of a generic European double-barrier option
- **KOC** : price of a knock-out call option
- **KOP** : price of a knock-out put option
- **KIC** : price of a knock-in call option
- **KIP** : price of a knock-in put option
- **UOC** : price of a up&out call option
- **DOC** : price of a down&out call option
- **UIC** : price of a up&in call option
- **DIC** : price of a down&in call option
- **UOP** : price of a up&out put option
- **DOP** : price of a down&out put option
- **UIP** : price of a up&in put option
- **DIP** : price of a down&in put option
- **OTH** : price of a one-touch option whose nominal amount is paid at the hit of the barrier level
- **OTE** : price of a one-touch option whose nominal amount is paid at the expiry of the contract
- **NT** : price of a no-touch option
- **DKOC** : price of a double-knock-out call option
- **DKOP** : price of a double-knock-out put option
- **DKIC** : price of a double-knock-in call option
- **DKIP** : price of a double-knock-in put option
- **DNT** : price of a double-no-touch option
- **DTE** : price of a double-touch option, paid at expiry
- **Fw**(t, T) : value of a forward contract (outright) at time t, expiring at time T
- **Fsw**(t, T) : value of an FX swap contract at time t, expiring at time T
- **STDL** : ATM straddle, i.e. a trading strategy (structure) involving the buying of a call and of a put struck at the same ATM level
- **RR** : risk reversal, i.e. a trading strategy (structure) involving the buying of a call against the selling of a put
- **VWB** : Vega-weighted butterfly, i.e. a trading strategy (structure) involving the buying of a strangle against the selling of an ATM straddle in such an amount as to make the total (BS model) Vega position nil
- **stdl** : ATM straddle price, in terms of BS implied volatility
- **RR** : risk reversal, i.e. a trading strategy (structure) involving the buying of a call against the selling of a put
- **rr** : risk reversal price, in terms BS of implied volatility

- **VWB** : Vega-weighted butterfly, i.e. a trading strategy (structure) involving the buying of a strangle against the selling of an ATM straddle in such an amount as to make the total (BS model) Vega position nil
- **vwb** : Vega-weighted butterfly price, in terms BS of implied volatility
- ATM : at-the-money level of the strike price of an option
- OTM : out-of-the-money level of the strike price of an option
- ITM : in-the-money level of the strike price of an option
- SDE : stochastic differential equation
- PDE : partial differential equation
- BS : Black&Scholes
- SV : stochastic volatility
- UV : uncertain volatility
- MIX : lognormal mixture