

The FX Market

The foreign exchange (FX) market is an OTC market where each participant trades directly with the others; there is no exchange, though we can identify some major geographic trading centres: London (the primary centre, where the primary banks' market makers are located; its importance has increased in the last few years), New York, Tokyo, Singapore and Sidney. This means that trading activity is carried out 24 hours a day, though in practice during London working hours the market has the most liquidity. Needless to say, the FX market experiences fierce competition amongst participants.

Most trades are currently carried out via interbank platforms (EBS is the most important). Anyway, the major market makers offer Internet platforms to their clients for quick trades and for leaving orders. The Reuters Dealing, which was the main platform in the past, has lately lost much of its pre-eminence. Basically, it is a chat system, connecting the participants, capable of recognizing the deal implicit in typical conversations between two professional operators, and transforming it into an automatic confirmation for the transaction. Nowadays, the Reuters Dealing is used mainly by option traders.

1.1 FX RATES AND SPOT CONTRACTS

Definition 1.1.1. FX rate. An exchange (FX) rate is the price of one currency in terms of another currency; the two currencies make a *pair*. The *pair* is denoted by a label, made up of two tags of three characters each: each currency is identified by its tag. The first tag in the exchange rate is the base *currency*, the second is the *numeraire currency*. So the FX is the price of the base currency in terms of the numeraire currency.

The numeraire currency can be considered as domestic: actually, in what follows we will refer to it as domestic. The base currency can be regarded as an asset whose trading generates profits and/or losses in terms of the domestic currency. In what follows the base currency will also be referred to as the foreign currency. We would like to stress that these denominations are not related to the perspective of the trader, who can actually be located anywhere and for whom the foreign currency may turn out to be indeed the domestic currency, from a "civil" point of view.

Example 1.1.1. The euro/US dollar FX rate is identified by the label EURUSD and it denotes how many US dollars are worth 1 euro. The domestic (numeraire) currency is the US dollar and the foreign (base) currency is the euro.

For each currency specific market conventions apply, and two of them are also important for the FX market: the *settlement date* and the *day count*. The settlement date (or delivery date) is the number of business days needed to actually transfer funds (if any are due) amongst interbank market participants after the closing of a deal; for most currencies it is two business days, but there are exceptions. In the market lore it is commonly referred to as "T + number of days", where "T" stands for the time (day) when the deal is closed. The *day count* is the

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Table 1.1 Settlement date and day count conventions for some major currencies

Tag	Currency	Settlement (T +)	Day count
AUD	Australian dollar	2	act/360
CAD	Canadian dollar	2	act/360
CHF	Swiss franc	2	act/360
CZK	Czech koruna	2	act/360
DKK	Danish krone	2	act/360
EUR	Euro	2	act/360
GBP	UK pound	0	act/365
HKD	Hong Kong dollar	2	act/365
JPY	Japanese yen	2	act/360
NOK	Norwegian kroner	2	act/360
NZD	New Zealand dollar	2	act/360
PLN	Polish zloty	2	act/360
SEK	Swedish krona	2	act/360
USD	US dollar	2	act/360
ZAR	South African rand	2	act/365

time factor used to calculate accrued interest between two dates in the money market of the relevant currency; it usually applies for simple compounding. A list of some currencies and their related settlement date and day count conventions is given in Table 1.1.

The settlement date and the day count for each currency are useful to price forward (outright) and FX swap contracts. There is a settlement date specific for the spot contract though, and it is the number of days, after the trade date, when the two amounts denominated in the currencies involved are exchanged between the counterparties. The rules to determine the settlement date for a spot contract are a little more complex, since they need the intersection of three calendars: we list them below when we define the spot contract.

The FX rates are expressed as five-digit numbers, with no regard for the number of decimals; the fifth digit is named *pip*: 100 pips make a *figure*. As an example, the major FX rates for spot contracts (we will define *spot* below) as of 29 October, 2007 are shown in Figure 1.1. Regular trades are for fixed amounts of the base currency. For example, if a trader asks for a spot price via the Reuters Dealing in the EURUSD, and they write

“I Buy (or Sell) 2 mios EURUSD at 1.3597”

this means that the trader buys (or sells) 2 million euros against 2 719 400 US dollars (1.3597×2 mios). Clearly, should one need exactly 1 million US dollars, it has to be specified as follows:

“I Buy 1 mio USD against EUR at 1.3597”

This means that the trader buys 1 million US dollars against 735 456 euros ($1/1.3597 \times 1$ million). The two contracts closed in the examples are *spot* and the employed FX rate is also said to be *spot*. We define the spot contract as follows:

Definition 1.1.2. Spot. Two counterparties entering into a spot contract agree to exchange the base currency amounts against an amount of the numeraire currency equal to the spot FX rate. The settlement date is usually two business days after the transaction date (but it depends on the currency).

GRAB CurrencyFXIA

FX Interest Rate Arbitrage Finder

Base Currency **EUR** Value Date **11/ 8/07**
 Swap Period **3.0** Month -or- Maturity Date **2/ 8/08**
 Number of Days **92** Today's Date **11/ 6/07**

ISO	Spot Rate	Outright	Fwd Points	Deposit	L	Arb. Rate	Basis
USD	1.45220	1.45378	0.00158	4.8750	L	4.4435	Act/360
EUR	1.00000	1.00000	0.00000	4.4435	U	4.4435	Act/360
JPY	166.68964	165.14898	-1.54065	0.8738	L	4.5323	Act/360
GBP	0.69653	0.69928	0.00275	6.2813	L	4.6300	Act/365
CHF	1.66457	1.65686	-0.00771	2.7500	L	4.5844	Act/360
CAD	1.34620	1.34752	0.00132	4.8350	L	4.4468	Act/360
AUD	1.57357	1.58367	0.01010	7.0175	L	4.4762	Act/360
NZD	1.87648	1.89624	0.01975	8.5800	L	4.4142	Act/360
HKD	11.28830	11.27265	-0.01565	4.0079	L	4.5017	Act/365
DKK	7.45480	7.45567	0.00087	4.8050	L	4.7585	Act/360
SEK	9.25240	9.25134	-0.00106	4.4750	L	4.5202	Act/360

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 4 24 7330 7500 Germany 49 69 920410
 Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
 0 06-Nov-07 12:18:54

Source: Bloomberg.

Figure 1.1 FX rates as of 29 October 2007.

As mentioned above, the settlement date for a spot contract is set according to specific rules involving three calendars (collapsing to two if the US dollar is one of the currencies of the traded pair). Here they are:

1. As a general rule, the settlement date for a spot contract is two business days after the trade date ($T + 2$), if this date is a business day for each of the two currencies of the pair. If this is not the case, the date is shifted forward until the condition is matched. An exception to this rule is the USDCAD (i.e., the US dollar/Canadian dollar pair), for which the settlement date is one business day after the trade date.
2. The settlement date set as in (1) must also be a business day in the USA, otherwise the date is shifted one day forward and the condition that the new date is a business day for each currency has to be checked again.
3. When the date after the trade date is a holiday in the USA (except for weekends), but not in other countries, then this date is counted as a business day to determine the settlement date. In this case it happens that for two days spot contracts will be settled on the same date, and in the market lore we say that the “settlement date is repeated”.

We provide an example to clarify how to actually apply these rules.

Example 1.1.2. Assume we are on Tuesday 20 November 2007; from market calendars it can be seen that Thursday 22 November is a holiday in the USA and Friday 23 November is a holiday in Japan. Consider three currencies: the US dollar, the euro and the yen. We consider the following possible trades with the corresponding settlement dates:

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- On 20 November we close a spot contract in EURUSD. The settlement date will be 23 November: two business days would imply 22 November, but this is a holiday in the USA, so the settlement date is shifted forward one day, a “good” business day for both currencies.
- On 21 November we close a spot contract in EURUSD. The settlement date will be 23 November (repeated): the holiday in the USA is one day after the trade and is not a weekend, so it is taken as a business day.
- On 20 November we close a spot contract in USDJPY. The settlement date will be 26 November: 22 November is a holiday in the USA, so the settlement date is shifted forward one day, but 23 November is a holiday in Japan, so the settlement date is shifted forward to the first available business day, which is Monday 26 November, after the weekend. The same calculation also applies if we traded in EURJPY.
- On 21 November we close a spot contract in USDJPY. The settlement date will be 26 November: 22 November is a holiday in the USA but it is taken as a business day; anyway, 23 November is a holiday in Japan but it is not counted as a business day, so the settlement date is shifted forward to the first available business day, which is Monday 26 November, after the weekend.
- On 22 November we close a spot contract in EURUSD; it is a US holiday but we can trade in other countries. The settlement date will be 26 November: 23 November is a “good” business day for both currencies, then there is the weekend, and Monday 26 November is the second business day.
- On 22 November we close a spot contract in EURJPY. The settlement date will be 27 November: 23 November is a good business day for the euro, but not for the yen, so we skip after the weekend, and Tuesday 27 November is the second business day, “good” for both currencies and the US dollar as well.

The rules for the calculation of the settlement date are probably the only real market-related technical issue a trader has to know, then they are ready to take part in the fastest game in town.

1.2 OUTRIGHT AND FX SWAP CONTRACTS

Outright (or forward) contracts are a simple extension of a spot contract, as is manifest from the following definition:

Definition 1.2.1. Outright. Two counterparties entering into an outright (or forward) contract agree to exchange, at a given expiry (settlement) date, the base currency amounts against an amount of the numeraire currency equal to the (forward) exchange rate.

It is quite easy to see that the outright contract differs from a spot contract only for the settlement date, which is shifted forward in time up to the expiry date in the future. That, however, also implies an FX rate, which the transaction is executed at, different from the spot rate and the problem of its calculation arises. Actually, the calculation of the forward FX price can easily be tackled by means of the following arbitrage strategy:

Strategy 1.2.1. Assume that we have an XXXYYY pair and that the spot FX rate is S_t at time t , whereas $F(t, T)$ is the forward FX rate for the expiry at time T . At time t , we operate the following:

- Borrow one unit of foreign currency XXX.

- Change one unit of XXX (foreign) against YYY and receive S_t YYY (domestic) units.
- Invest S_t YYY in a domestic deposit.
- Close an outright contract to change the terminal amount back into XXX, so that we receive $S_t \frac{1}{P^d(t, T)} \frac{1}{F(t, T)} \text{XXX}$.
- Pay back the loan of one YYY plus interest.

To avoid arbitrage, the final amount $S_t \frac{1}{P^d(t, T)} \frac{1}{F(t, T)} \text{XXX}$ must be equal to the value of the loan of 1 XXX at time T , which can be calculated by adding interest to the notional amount.

This strategy can be translated into formal terms as:

$$S_t \frac{1}{P^d(t, T)} \frac{1}{F(t, T)} = 1 \frac{1}{P^f(t, T)}$$

which means that we invest the S_t YYY units in a deposit traded in the domestic money market, yielding at the end $S_t \frac{1}{P^d(t, T)}$ ($P^d(t, T)$ is the price of the domestic pure zero-coupon bond), and change then back to XXX currency at the $F(t, T)$ forward rate. This has to be equal to 1 XXX units plus the interest prevailing in the foreign money market ($P^f(t, T)$ is the price of the foreign pure zero-coupon bond). Hence:

$$F(t, T) = S_t \frac{P^f(t, T)}{P^d(t, T)} \quad (1.1)$$

In Chapter 2 we will see an alternative, and more thorough, derivation for the fair price of a forward contract. The FX rate in equation (1.1) is that which makes the value of the outright contract nil at inception, as it has to be since no cash flow from either party is due when the deal is closed.

A strategy can also be operated by borrowing money in the domestic currency, investing it in a foreign deposit and converting it back into domestic currency units by an outright contract. It is easy to see that we come up with the same value of the fair forward price as in equation (1.1), which prevents any arbitrage opportunity.

The careful reader has surely noticed that in Strategy 1.2.1 the prices of pure discount bonds have been used to calculate the present and future value of a given currency amount. Actually, the market practice is to use money market conventions to price the deposits and hence to determine the forward FX rates. The use of pure discount bonds (also known as discount factors) is perfectly consistent with the market methodology as long as they are derived by a *bootstrap* procedure from the available market prices of the deposits.

Remark 1.2.1. Strategy 1.2.1 is model-independent and operating it carries the forward price $F(t, T)$ at a level consistent with the other market variables (i.e., the FX spot rate and the domestic and foreign interest rates), so any arbitrage opportunity is cleared out. It should be stressed that two main assumptions underpin the strategy: (i) counterparties are not subject to default risk, and (ii) there is no limit to borrowing in the money markets.

Assume that the first assumption does not hold. When we invest the amount denominated in YYY in a deposit yielding domestic interest, we are no longer sure of receiving the amount $S_t \frac{1}{P^d(t, T)}$ at time T to convert back into XXX units since the counterparty, to whom we lent money, may go bankrupt. We could expect to recover a fraction of the notional amount of the deposit, but the strategy is no longer effective anyway. In this case we may have a forward price

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1 $F(t, T)$ trading in the market which is different from that determined univocally by Strategy
2 1.2.1, and we cannot operate the latter to exploit an arbitrage opportunity, since we would
3 bear a risk of default that is not considered at all.

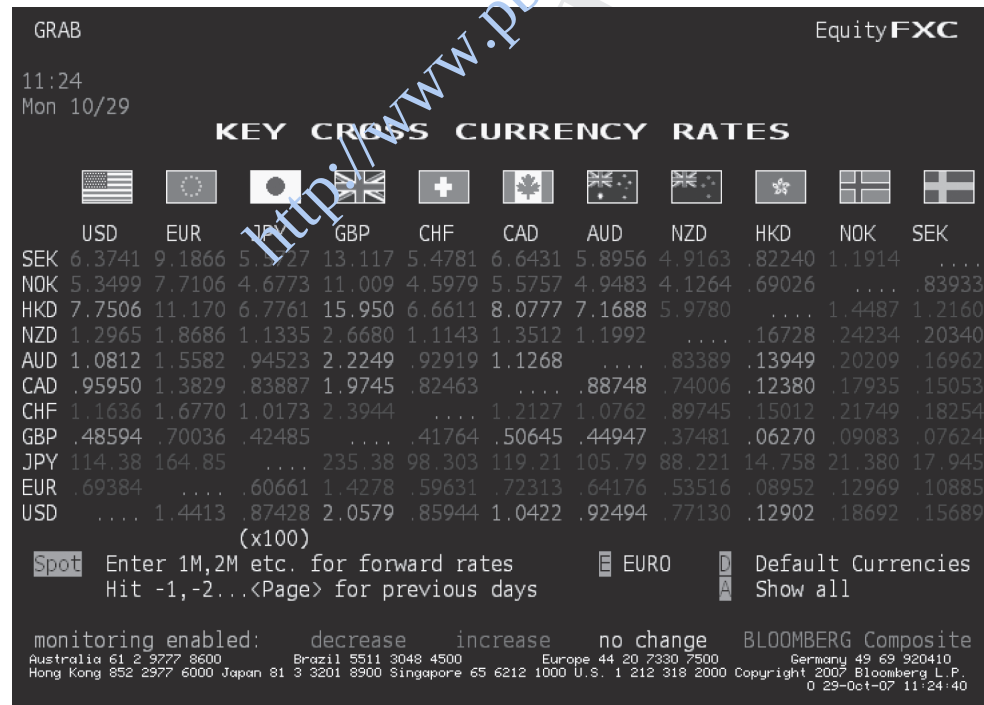
4 Assume now that the second assumption does not hold. We could observe a forward price
5 in the market higher than that determined by Strategy 1.2.1, but we are not able to exploit the
6 arbitrage opportunity just because there is a limited amount of lending in the market, so we
7 cannot borrow the amount of one unit of XXX currency to start the strategy.

8 In reality, both situations can be experienced in the market and actually the risk of default
9 can also strongly affect the amount of money that market operators are willing to lend amongst
10 themselves. Starting from July 2006, a financial environment with a perceived high default
11 risk related to financial institutions and a severe shrinking of the available liquidity has been
12 very common, so that arbitrage opportunities can no longer be fully cleared out by operating
13 the replication Strategy 1.2.1.

14 In the market, outright contracts are quoted in forward points:

$$\mathbf{Fpts}(t, T) = F(t, T) - S_t$$

15 Forward points are positive or negative, depending on the interest rate differentials, and they
16 are also a function of the level of the spot rate. They are (algebraically) added to the spot rate
17 when an outright is traded, so as to get the fair forward FX rate. In Figure 1.2, forward points
18
19
20
21
22



Source: Bloomberg.

Figure 1.2 Forward points at 6 November 2007.

at 6 November 2007 for a three-month delivery are shown – they are the same points used in FX swap contracts, which will be defined below. The base currency is the euro and forward points are referred to each (numeraire) currency listed against the euro: in the column “Arb. rate” the forward implied no-arbitrage rate for the euro is provided and it is derived from the formula to calculate the forward FX rate so as to match the market level of the latter.

For the sake of clarity and to show how forward FX rates are actually calculated, we provide the following example:

Example 1.2.1. Assume we have the market data as in Figure 1.2. We want to check how the forward points for the EURUSD are calculated. We use formula (1.1) to calculate the forward FX rate, but we apply the money market conventions for capitalization and for discounting (i.e., simple compounding):

$$F(0, 3M) = 1.4522 \frac{\left(1 + 4.875\% \frac{92}{360}\right)}{\left(1 + 4.4435\% \frac{92}{360}\right)} = 1.45378$$

where 3M stands for “three-month expiry”. Hence, the FX swap points are calculated straightforwardly as:

$$\text{Fpts}(0, 3M) = F(0, 3M) - S_0 = 1.45378 - 1.4522 = 0.00158$$

so that both the forward FX rate and forward points are verified by what is shown in the figure.

The FX swap is a very popular contract involving a spot and an outright contract:

Definition 1.2.2. FX swap. Two counterparties entering into an FX swap contract agree to close a spot deal for a given amount of the base currency, and at the same time they agree to reverse the trade by an outright (forward) with the same base currency amount at a given expiry.

From the definition of an FX swap, the valuation is straightforward: it is the sum of a spot contract and the value of a forward contract. So, we just need the spot rate and the forward points, which are denominated (FX) swap points when referred to such a contract. A typical request by a trader on the Reuters Dealing (which is still one of the main platforms where FX swap contracts can be traded) might be:

“I buy and sell back 1 mio EUR against USD in 3 months”

This means that the trader enters into a spot contract buying 1 million euros against US dollars, and then sells then back at the expiry of the FX swap in three months time. We use market data provided in the Bloomberg screen shown in Figure 1.2 to see, in practice, how the FX swap contract implied by the request above is quoted and traded. Besides, in the example the difference between a *par* (alternatively an *even*) FX swap and a *non-par* (alternatively an *uneven* or *split* or *change*) FX swap is stressed.

Example 1.2.2. We use the same market data as in Example 1.2.1 and in Figure 1.2. The current value of a 3M FX swap “buy and sell back 1 mio EUR against USD” has to be split

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1 into its domestic (US dollar in our case) and foreign (euro) components:

$$\begin{aligned}
 \mathbf{Fsw}^d(0, 3M) &= -S_0 + \frac{1}{(1 + r^d \tau)} F(0, 3M) \\
 &= -1.4522 + 1.45378 \frac{1}{\left(1 + 4.875\% \frac{92}{360}\right)} = -0.0163 \text{ USD} \\
 \mathbf{Fsw}^f(0, 3M) &= 1 - \frac{1}{(1 + r^f \tau)} \\
 &= 1 - \frac{1}{\left(1 + 4.4435\% \frac{92}{360}\right)} = 0.0112 \text{ EUR}
 \end{aligned}$$

13 In the two formulae above we just calculated the present value for all the cash flows provided
14 by the FX swap contract, separately for each of the two currencies involved. An outflow of S_0
15 US dollars against 1 euro at inception and an inflow of $F(0, 3M)$ on the delivery date against 1
16 euro again. The two final values are expressed for each leg of the corresponding currency. This
17 is a par FX swap contract, since the notional amount (1 million euros) exchanged at inception
18 via the spot transaction, and the final amount exchanged back at expiry, via the outright
19 transaction, are the same. It is manifest that a par FX swap engenders a position different from
20 0 in both currencies. Professional market participants prefer to have nil currency exposure (we
21 will see why later), so they prefer to trade non-par FX swaps. In this trade the amount of the
22 base currency exchanged at the forward expiry is modified so as to generate a zero currency
23 exposure. It is easy to see that the amount to be exchanged (so as to have a par FX swap)
24 has to be compounded at the numeraire (foreign) currency interest rate. Hence, if we set the
25 amount of euros to be exchanged on the delivery date equal to $(1 + 4.4435\% \frac{92}{360}) = 1.0114$
26 instead of 1, we get:

$$\begin{aligned}
 \mathbf{Fsw}^d(0, 3M) &= -S_0 + \frac{(1 + r^f \tau)}{(1 + r^d \tau)} F(0, 3M) \\
 &= -1.4522 + 1.45378 \frac{\left(1 + 4.4435\% \frac{92}{360}\right)}{\left(1 + 4.875\% \frac{92}{360}\right)} = 0 \text{ USD} \\
 \mathbf{Fsw}^f(0, 3M) &= 1 - \frac{(1 + r^f \tau)}{(1 + r^f \tau)} \\
 &= 1 - 1 = 0 \text{ EUR}
 \end{aligned}$$

39 which clearly shows no residual exposure to the FX risk.

40 The quoted price of an FX swap contract will be simply the forward points. They are related
41 to the FX spot level, to be specified when closing the contract. When uneven FX swaps are
42 traded, the domestic interest rate has to be agreed upon as well.

43 After this short analysis, we are able to sum up the specific features of outright and FX
44 swap contracts:

- 45 1. An outright contract is exposed to an FX rate risk for the full nominal amount. It also has
46 exposure to interest rates, although this is very small compared to the FX risk.
47

2. In an FX swap contract the FX rate risk of the spot transaction is almost entirely offset by the outright transaction. In the case of non-par contracts, the FX risk is completely offset, and only a residual exposure to the interest rate risk is left.
3. For the reasons above, outright contracts are mainly traded by speculators and hedgers in the FX market.
4. The FX swap is rather a treasury product, traded in the interbank market to move funds from one currency to another, without any FX risk (for par contracts), and to hedge or get exposure to the interest rate risks in two different currencies. Nonetheless, it is used by options traders to hedge exposure to the domestic and foreign interest rates.

Remark 1.2.2. *If we assume that we are working in a world where the occurrence of default of a counterparty is removed, then by standard arbitrage arguments we must impose that the forward points of an outright contract are exactly the same as the swap points of an FX swap contract. Things change if we introduce the chance that market operators can go bankrupt, so that the mechanics of the two contracts imply great differences in their pricing.*

We have seen before that the arbitrage argument of the replica Strategy 1.2.1 can no longer be applied when default is taken into account, so that the actual traded forward price can differ substantially from the theoretical arbitrage price, since a trader can suffer a big loss if the counterparty from whom they bought the deposit defaults. Now, we would like to examine whether removing the no-default assumption impacts in the same way both the outright and the FX swap contract.

To this end, consider the case when the FX swap points for a given expiry imply a tradable forward price $F'(t, T)$ greater than the theoretical price $F(t, T)$ obtained by formula (1.1). To exploit the possible arbitrage, we could borrow one million units of foreign currency, say the euro, and close an FX swap contract “sell and buy back 1 mio EUR, uneven amount”, similar to that in Example 1.2.2, but with a reverse sign. Basically, we are operating Strategy 1.2.1 with an FX swap, instead of an outright contract. Assume also that, after the deal is struck, our counterparty in the FX swap deal might be subject to default, in which case they will not perform their contractual obligations, so we will not receive back the one million euros times $(1 + r^f \tau)$, against $F'(t, T)$ million US dollars times $(1 + r^f \tau)$ paid by us. In such an event, we will not have the amount of money we need to pay back our loan in euros, whose value at the end of the contract is equal to $(1 + r^f \tau)$ million euros. Nevertheless, we still have the initial exchanged amount in USD, equal to S_t (the FX spot rate at inception of the contract), and we could use this to pay back our debt. In this case, assuming we have kept the amount in cash, we can convert it back into euros at the terminal FX spot rate S_T , which might be lower or higher than S_t , so that we can end up with a final amount of euros greater or smaller than one million (the euro amount will be S_t/S_T). The terminal economic result could be a profit or a loss, depending on the level of the FX spot rate S_T and on how much we have to pay for the interest on the loan in euros. Nonetheless, we may reasonably expect not to lose as much as one million euros, and the total loss (or even profit) is a function of the volatility of the exchange rate and the time to maturity of the contract.

Assume now that we operated Strategy 1.2.1 with an outright contract. We borrow one million euros, convert it into dollars at S_t , buy a deposit in dollars, and convert the terminal amount by selling an outright at the rate $F'(t, T)$. If our counterparty defaults, they will not pay back the amount of money we lent to them (supposing there is no fraction of the notional amount recovered) and we will end up with no money to sell via the outright, so as to convert it into euros and pay back our loan. In this case we are fully exposed to the original amount

of one million euros and we will suffer a loss for sure equal to this amount, plus the interest on the loan.

From the two cases we have described, we can see that the FX swap can be considered as a collateralized loan. The example shows a situation just as if we lent an amount denominated in euros, collateralized by an amount denominated in dollars. Clearly, the collateral is not risk-free, since its value in euros is dependent on the level of the exchange rate, but it is a guarantee that will grant a presumably high recovery rate of the amount lent on the occurrence of default of the counterparty, and we could possibly end up with a profit. In the other case we examined, that is the outright contract, we see that we have no collateral at all as a guarantee against the default of the counterparty, so we are fully exposed to the risk of losing the amount of dollars we lent to them. This loss can be mitigated if we assume that we can recover a fraction of the notional amount we lent, but the recovery will very likely be much smaller than the fraction of notional we can recover via the collateral.

There are two conclusions we can draw:

1. The forward rate $F(t, T)$ determined as in equation (1.1) does not identify the unique arbitrage-free price of an outright contract, if we include the chance of default of the counterparty.
2. The forward price implied by an FX swap contract can be different from that of an outright contract when default of the counterparty is considered, because Strategy 1.2.1 operated with an FX swap is less risky than the same strategy operated with an outright contract.

1.3 FX OPTION CONTRACTS

FX options are no different from the usual options written on any other asset, apart from some slight distinctions in the jargon. The definition of a *plain vanilla European* option contract is the following:

Definition 1.3.1. European plain vanilla FX option contract. Assume we have the pair XXXYYY. Two counterparties entering into a plain vanilla FX option contract agree on the following, according to the type of option traded:

- **Type XXX call YYY put:** the buyer has the right to enter at expiry into a spot contract to buy (sell) the notional amount of the XXX (YYY) currency, at the strike FX rate level K .
- **Type XXX put YYY call:** the buyer has the right to enter at expiry into a spot contract to sell (buy) the notional amount of the XXX (YYY) currency, at the strike FX rate level K .

The spot contract at expiry is settled on the settlement date determined according to the rules for spot transactions. The notional amount N in the XXX base currency is exchanged against $N \times K$ units of the numeraire currency. The buyer pays a premium at inception of the contract for their right.

The following chapters are devoted to the fair calculation of the premium of an option, the analysis of the risk exposures engendered by trading it, and the possible approaches to hedging these exposures. Clearly, this will be done not only for plain vanilla options, but also for other kinds of options, usually denoted as *exotics*. A very rough taxonomy for FX options is presented in Table 1.2, this should be considered just as a guide to how the analysis will be organized in what follows. Besides, it is worth noticing that the difference between

Table 1.2 Taxonomy of FX options.

Group	Name	Exercise	Monitoring
Plain-vanilla	Call/put	E/A	E
First-generation exotic	Digital	E	E
First-generation exotic	Knock-in/out barriers	E/A	E/C/D
First-generation exotic	Double-knock in/out barriers	E/A	E/C/D
First-generation exotic	One-touch/no-touch/ double-no-touch/double-touch	A	C/D
First-generation exotic	Asian	E/A	D
First-generation exotic	Basket	E/A	D
Second-generation exotic	Window knock-in/out barriers	E/A	E/C/D
Second-generation exotic	First-in-then-out barriers	E/A	E/C/D
Second-generation exotic	Forward start plain/barriers	E/A	E/C/D
Second-generation exotic	External barriers	E/A	E/C/D
Second-generation exotic	Quanto plain/barriers	E/A	E/C/D

Exercise: European (E), American (A). Monitoring: at expiry (E), continuous (C), discrete (D).

first-generation and second-generation exotics is due to the time sequence of their appearance in the market rather than any reference to their complexity.

It is worth describing in more detail the option contract and the market conventions and practices relating to it.

1.3.1 Exercise

The exercise normally has to be announced by the option's buyer at 10:00 AM New York time; options are denominated *NY Cut* in this case, and they are the standard options traded in the interbank market. The counterparties may also agree on a different time; such as 3:00 PM Tokyo time; in this case we have the *Tokyo Cut*. The exercise is considered automatic for a given percentage of in-the-moneyness of the options at expiry (e.g., 1.5%), according to the ISDA master agreement signed between two professional counterparties before starting any trading activity between them. In other cases the exercise has to be announced explicitly, although it is market fairness to consider exercised (or abandoned) options manifestly in-the-money (or out-of-the money), even without any call from the option's buyer.

1.3.2 Expiry date and settlement date

The expiry date for an option can be any date when at least one marketplace is open, then the settlement date is set according to the settlement rules used for spot contracts. Some market technicalities concern the determination of the expiry and settlement (delivery) dates for what we call *canonic* or *standard* dates. In more detail, in the interbank market daily quotes are easily available for standard expiries expressed in terms of time units from the trade date, i.e., overnight, weeks, months and years.

Day periods. Overnight is the simplest case to analyse, since it indicates an expiry for the next available business day, so:

1. In normal conditions it is the day after the trade date or after three days in case the trade date is a Friday (due to the weekend).

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2. The expiry is shifted forward if the day after the trade date is not a business day all around the world (e.g., 25 December). On the contrary if at least one marketplace is open, then the expiry date is a good one.
3. Once the expiry date is determined, the settlement date is calculated with the rules applied for the spot contract.

If the standard expiry is in terms of number of days (e.g., three days), the same procedure as for overnight applies, with expiry date initially and tentatively set as the number of days specified after the trade date.

Week periods. This case is not very different from the day period one:

1. The expiry is set on the same week day (e.g., Tuesday) as the trade date, for the given number of weeks ahead in the future (e.g., 2 for two weeks).
2. At least one marketplace must be open, otherwise the expiry is shifted forward by one day and the open market condition checked again.
3. Once the expiry is determined, the usual rules for the spot contract settlement date apply.

Month and year periods. In these cases a slightly different rule applies, since the spot settlement date corresponding to the trade date is the driver. More specifically:

1. One moves ahead in the future by the given number of periods (e.g., 6 for six months), then the same day of the month as the spot settlement date (corresponding to the trade date, in the current month) is taken as the settlement date of the option (e.g., again for six-month expiry, if the trade date is the 13th of the current month and the 15th is the settlement date for a corresponding spot contract, then the 15th day of the sixth month in the future will be the option settlement date). If the settlement date of the future month is not a valid date for the pair involved, then the date is shifted forward until a good date is achieved.
2. If the settlement determined in (1) happens to fall in the month after the one corresponding to the number of periods considered (e.g., the six-month expiry yields a settlement actually falling in the seventh month ahead), then the *end-of-month* rule applies. From the first settlement date (identified from the spot settlement of the trade date), the date is shifted backward until a valid (for the contract's pair) settlement date is reached.
3. The expiry can now be calculated by applying backward from the settlement date the rules for a spot contract.
4. The year period is treated with same rules simply by considering the fact that one year equals 12 months.

We provide an example to clarify the rules listed above.

Example 1.3.1. Assume we trade an option EUR call USD put with expiry in one month. We consider the following cases:

- The trade date is 19 October 2007. From the market calendars the spot settlement date for such a trade date can be calculated and set on 23 October so that the settlement of the option has to be set on 23 November (i.e., the same day one month ahead). This date can be a settlement date for the EURUSD pair and the corresponding expiry date is 21 November, since the 22nd is a holiday in the USA but is counted as a business day according to the spot date rules. Actually, we know from Example 1.1.2 that the spot trades dealt on 20 November also imply a settlement date on the 22nd. When the expiry date is calculated

working backward from the settlement, the first possible trade date encountered is taken (i.e., the 21st in this case).

- The trade date is 19 October 2007. From the market calendars the spot settlement date for such a trade date is 24 October, thus the option's settlement date is 24 November, which is a Saturday, so it is shifted forward to the first available business day for both currencies: Monday 26 November. Working backward to calculate the expiry date, we would take 22 November but this is a US holiday, so we move one more day backward and set the expiry on the 21st, which agrees with spot settlement rules.

After analysing the rules for standard expiries, for the sake of completeness we just remark that if a specific date is agreed upon for the expiry (e.g., 7 January 2008), then the standard spot settlement rules apply to calculate the option's settlement date (9 January, if the contract's pair is EURUSD).

1.3.3 Premium

The option's premium is paid on the spot settlement date corresponding to the trade date. It can be paid in one of either currencies of the underlying pair and it can be expressed in four different ways, which we list below:

1. *Numeraire currency units* (p_{numccy}). This is the standard way in which, for some pairs, premiums are expressed for plain vanilla options in the interbank market after the closing of the deal. It is worth noticing also that this is the natural premium one calculates by a pricing formula. The actual premium to pay is calculated by multiplying the currency units times the notional amount (in base currency units): $N \times p_{numccy}$.
2. *Numeraire currency percentage* ($p_{numccy\%}$). This is the standard way in which premiums are expressed and quoted for exotic (one-touch, double-no-touch, etc.) options in the interbank market, when the payout is a numeraire currency amount. It can be calculated by dividing the premium in numeraire currency units by the strike: $p_{numccy\%} = \frac{p_{numccy}}{K} \times 100$. The actual premium to pay is equal to the notional amount in numeraire currency units ($N \times K$) times the numeraire currency percentage premium: $N_{numccy} \times \frac{p_{numccy\%}}{100}$.
3. *Base currency units* ($p_{baseccy}$). This way of quoting may be useful when the numeraire currency amount is fixed for all the options entering into a given strategy (e.g., in a EUR call USD put spread). It can be calculated by dividing the premium in numeraire currency units by the spot FX rate and then by the strike: $p_{baseccy} = \frac{p_{numccy}}{S_t K}$. The actual premium to pay is equal to the notional amount, expressed in numeraire currency (that is: $N \times K$), times the base currency units premium: $N_{numccy} \times p_{baseccy}$.
4. *Base currency percentage* ($p_{baseccy\%}$). This is the standard way in which premiums are expressed and quoted for exotic (barrier) options, and for some pairs also for plain vanilla options, in the interbank market. It can be calculated by dividing the premium in numeraire currency units by the spot FX rate: $p_{baseccy\%} = \frac{p_{numccy}}{S_t} \times 100$. The actual premium to pay is equal to the notional amount times the base currency percentage premium: $N \times \frac{p_{baseccy\%}}{100}$.

In Table 1.3 we report some market conventions for option premiums; usually, the numeraire currency premium is multiplied by a factor such that it is expressed in terms of pips (see above for the definition of the latter), or as a percentage of either notional rounded to the nearest quarter of 0.01%. We will see later that the way markets quote premiums has an impact on the building of the volatility matrix, so that it is not just a curiosity one may lightly neglect.

Table 1.3 Market conventions for option premiums for some pairs

Pair	p_{numccy}	$p_{baseccy}\%$
EURUSD	USD pips	
EURCAD	CAD pips	
EURCHF		EUR %
EURGBP	GBP pips	
EURJPY		EUR %
EURZAR		EUR %
GBPCHF		GBP %
GBPJPY		GBP %
GBPUSD	USD pips	
USDCAD		USD %
USDCHF		USD %
USDJPY		USD %
USDZAR		USD %

Example 1.3.2. Assume we want to buy 2 000 000 EUR call USD put struck at 1.3500, with a reference EURUSD spot rate equal to 1.2800. The notional amount in USD is $2\,000\,000 \times 1.3500 = 2\,700\,000$. The premium can be quoted in one of the four ways we have examined and we have that:

1. If the premium is in numeraire currency units and it is $p_{USD} = 0.0075$ US dollars per one EUR unit of option, we will pay $2\,000\,000 \times 0.0075 = 15\,000$ USD.
2. If the quotation is expressed as a numeraire currency percentage, the premium is $p_{USD\%} = \frac{0.0075}{1.3500} \times 100 = 0.5550\%$ (rounded to the nearest quarter of 0.01%) for one USD unit of option dollar, and we pay $0.5550 \times \frac{2\,700\,000}{100} = 14\,985$ USD (the small difference of 15 000 is due to rounding conventions).
3. If the quotation is in base currency units, the premium is $p_{EUR} = \frac{0.75}{1.2800 \times 1.3500} = 0.00435$ EUR per one USD unit of option dollar, and we pay $0.55 \times \frac{2\,700\,000}{100} = 11\,750$ EUR.
4. Finally, if the premium is expressed as a base currency percentage, it is $p_{EUR\%} = \frac{0.0075}{1.2800} \times 100 = 0.5875\%$ of the EUR notional (rounded to the nearest quarter of 0.01%) and we pay $0.5875 \frac{2\,000\,000}{100} = 11\,750$ EUR.

1.3.4 Market standard practices for quoting options

FX options can be dealt for any expiry and also for any level of strike price. Amongst professionals, options are quoted according to standards: some of them are actually rather clever, and make FX options one of the most efficient OTC derivatives markets.

Let us start with plain vanilla options. Firstly, options are usually quoted for standard dates, although it is possible to ask a market maker for an expiry occurring on any possible date. Secondly, quotations are not in terms of (any of the four above) premiums but in terms of implied volatilities, that is to say, in terms of the volatility parameter to plug into the BS model (given the values of all the other parameters and the level of the FX spot rate, retrievable from the market). Once the deal is closed, the counterparties may agree to actually express the premium in any of the four ways listed above, although the standard way is in numeraire

currency pips (p_{numccy}). Thirdly, strike prices are quoted in terms of the Delta¹ of the option: this means that before closing the deal, the strike level is not determined yet in absolute terms. Once the deal is closed, given the level of the FX spot rate and the implied volatility agreed upon (the interest rate levels will be taken from the money market), the strike will be set at a level yielding the BS Delta the two counterparties were dealing. This way of quoting is smart: it allows us not to worry about small movements of the underlying market during the bargaining process, because the absolute strike level will be defined only after the agreement on the price (in terms of implied volatility), so that the trader is sure to trade an option with given features in terms of exposures both to the underlying pair and to the implied volatility.²

If not otherwise specified when asking for a quote, the option is considered to be traded Delta-hedged (“with Delta exchange”), i.e., a spot trade offsetting the BS Delta exposure is closed along the option’s transaction. Usually, for strikes very far OTM with a very tiny premium (p_{numccy}) and a negligible Delta exposure, options are quoted at an absolute level of premium and with no Delta hedge (“without Delta exchange”).

For popular exotic options³ some other conventions are in force for ordinary market activity. For barrier options, contrary to plain vanilla options, when a trader asks for a price, strikes and barrier levels are asked for in absolute terms, by specifying the reference spot FX rate, and also an ATM implied volatility level. The quote will be assumed to be valid for those levels, and it will be provided in terms of the premium as a percentage of the base currency notional. Also, for barrier options it is assumed that the deal includes a Delta-hedge transaction and in most cases a Vega-hedge⁴ transaction (by dealing a spot contract and an ATM straddle⁵ to offset the related exposures). The amounts dealt in those transactions are calculated according to the BS model, using as inputs the reference FX spot and implied volatility levels.

Other very common exotics are the bet options,⁶ i.e., one-touch, no-touch, double-no-touch, double-touch, digitals. They are quoted as a percentage of the notional amount (which is the payout of the bet, usually in base currency), given reference levels of the FX spot and implied volatility. After the agreement on the price, the deal will include the Delta-hedge and Vega-hedge transactions (to be defined according to the BS model).

In the following example we provide some customary conversations between professional traders. We just mean to clarify the conventions we have described above, and are aware that we are anticipating many of the issues that will be investigated in detail in the following chapters. So, the reader should not be worried if they feel somewhat lost.

Example 1.3.3. *On the Reuters Dealing, which we have already mentioned to be the main trading platform for FX, options are traded via conversations like those below:*

- *Plain vanilla*
 - > *Please, 3M EUR call USD put 25D, in 30.*
 - > *7.5 7.7*
 - > *7.7 pls, spot ref 1.4575.*

¹ The Delta of an option will be defined in Chapter 2, where the BS model is presented.

² This statement will become clearer with the analysis in Chapter 3.

³ The definition for each of the options we mention below will be given in the chapters devoted to their analysis.

⁴ Vega will be defined in Chapter 2.

⁵ This structure is described later on in this chapter.

⁶ More details about the definition of bet options can be found in Chapter 6.

The first trader asks for a price for EUR call USD put expiring in three months with a strike level not yet defined in absolute terms, but referred to in terms of 25% Delta EUR call, with a notional amount of 30 million euros. The second trader quotes a bid/ask in terms of BS implied volatility and the first trader is buying the options paying 7.7% and providing also the FX spot level (set reasonably near to the market level), which will be used to calculate the strike level corresponding to the 25% EUR call, the premium (in USD pips) and will also be the level of the Delta-hedge transaction. In fact, since there was no mention of it in the request for the quote, it is assumed to be included in the deal.

• **Barrier option**

> Please, 6m EUR put USD call 1.4500 RKO 1.3800, spot ref 1.4576, in 50 with VH.
> 0.20 0.25
> 0.25 pls.

The first trader asks for a quote in a RKO barrier EUR put USD call expiring in six months. The strike (1.4500) and the barrier (1.3800) are specified right from the start of the request. The notional amount is 50 million euros and the asked quote is for a trade including the Vega hedge ("with VH"), besides the Delta hedge. The second trader's quote is in absolute premiums, in terms of a percentage of the notional amount, so that when the first trade accepts to buy by applying the offer, they will pay 0.25% of 50 million euros.

• **Double-no-touch**

> Please, 1Y EURUSD DNT 1.3500 1.4500, in 1 mio EUR with VH.
> 20 25
> 20 pls.

The first trader asks for a quote in a double-no-touch expiring in 1Y on the EURUSD pair, with lower range level at 1.3500 and upper range level at 1.4500. The payout is in 1 million euros and the trade will include the Vega hedge. The second trader's quote is in absolute premium, expressed as a percentage of the payout, so that the first trader will cash in 200 000 euros since they are selling the options by applying the bid (20%).

1.4 MAIN TRADED FX OPTION STRUCTURES

Although the FX option market is very liquid for options with any kind of strike level and expiry, nonetheless it is possible to identify some structures that are very popular amongst professional market participants. We will understand why later on, when we examine how to manage the volatility risk of an options portfolio, and we will also study the features and behaviour of their risk exposure.

The first structure is the ATM *straddle* (STDL hereafter): that is, the sum of a (base currency) call and a (base currency) put struck at the at-the-money level. The quotes for this structure on standard expiries are the most liquid ones.

One has to pay some attention when defining the exact strike the market is referring to in trading ATM options, since several definitions exist. The first kind of ATM is the *at-the-money spot*: in this case, the strike of the option is set equal to the FX spot rate; the expiry is immaterial in determining the strike. The second kind is the ATM *forward*: the strike is set equal to the forward price of the underlying pair for the same expiry of the option; in this case, we have different ATM strikes for each maturity (recall formula (1.1)). The third kind is the 0 *Delta STDL*: the strike is chosen so that, given the expiry, a put and a call have the same Delta but with different signs. This implies that no Delta hedge is needed when trading the straddle. We will see later how to retrieve this strike. The ATM implied volatility quoted in the FX option

market is the one referring to a 0 Delta **STD**L strike, and hence it is the implied volatility to plug into the BS formula when trading an ATM **STD**L.

The amount of an ATM **STD**L is traded as the sum of the (base currency) amounts of two component options.

Example 1.4.1. Suppose we want to buy an ATM **STD**L. On the Reuters Dealing we can ask a broker or a market maker for this structure, and can experience a conversation like the following:

> Please, 1M EURUSD ATM straddle in 50.
> 8.10 8.30
> 8.30 pls, spot ref 1.4575.

The first trader asks for an ATM **STD**L in 50 million EUR, meaning that if the deal is struck they will trade in a straddle made up of 25 million EUR put and 25 million EUR call. The words “ATM straddle” are actually redundant, since “1M in 50” will unequivocally indicate an ATM **STD**L. The second trader makes a quote and the first trader applies the offer at 8.30%, thus buying the structure, suggesting also the reference level for the FX spot rate at 1.4575, upon which the ATM strike will be set and the premium is calculated by using the dealt implied volatility (8.30%). Clearly, by definition, no Delta hedge will be exchanged since the **STD**L will engender no exposure to the FX rate.

Besides the ATM **STD**L, there are at least two other structures frequently traded: they are the 25% Delta risk reversal (**RR** hereafter) and the 25% Delta Vega-weighted butterfly (**VWB** hereafter).⁷

The **RR** is a structure set up when one buys a (base currency) call and sells a (base currency) put both featured with a symmetric Delta (long **RR**), or the reverse (short **RR**). Delta can be chosen equal to any level, but the 25% is the most liquid one so that the call and the put entering into the **RR** will have a strike level yielding a 25% Delta, without considering its sign (actually, for puts it will be negative). The **RR** is quoted as the difference between the two implied volatilities to plug into the BS formula in order to price two legs of the structure, and we indicate this price in volatility as **rr**. A positive number means that the call is favoured and that its implied volatility is higher than the implied volatility of the put; a negative number implies the opposite. For example, if the three-month 25% Delta **rr** for the EURUSD pair is −0.5%, then the implied volatility of the EUR call is 0.5% lower than the EUR put (both struck at a level yielding 25% Delta, without considering the sign). At time t , we can write the price (in implied volatility terms) of a 25% Delta **RR** with maturity in T as:

$$\mathbf{rr}(t, T; 25) = \sigma_{25C}(t, T) - \sigma_{25P}(t, T) \quad (1.2)$$

where $\sigma(t, T)$ is the implied volatility at t for an option expiring in T and struck at the level indicated in the subscript.

The amount of a **RR** is typically denominated in terms of base currency units, and it is referred to the amount of base currency call that will be traded against the equal amount of base currency put.

⁷ It is worth noticing here that in market lore traders refer to options struck at a level implying a 25% Delta, without considering the sign, indistinctly as a 25 or 0.25 or finally 25% Delta call or put.

Example 1.4.2. We present a market conversation to deal a 25 Delta **RR**:

> Please, usdjpy 6M 25D RR in 100.
> 1.70 1.80 P
> 1.70 pls, spot ref 108.35.
> OK, vols 11.85 10.15

The first trader asks for a 25 Delta risk reversal in USDJPY, in an amount of 100 million US dollars. If the deal is closed, it will be traded in 100 million USD call JPY put against 100 million USD put JPY call. The second trader makes a quote and the deal is struck because the first trader hits the bid at 1.70%. The **rr** is favouring the USD put, as indicated by the “P” after the quotes. This is usually disregarded amongst professionals when there is no possibility of misunderstanding (as in this case, where the **rr** is far from 0 and the market makers are supposed to know what type of options are favoured). The suggestion of the reference for the USDJPY spot rate at 108.35, if accepted, will allow us to determine the strikes corresponding to the 25 Delta USD call and USD put, by also using the two volatilities. These are determined starting from the ATM level dealing in the market when the **RR** is closed, and then adding half the dealt price for the **RR** (0.85% in the example) from the USD put since it is favoured, and subtracting half the price from the USD call. Should the USD call be favoured instead of the put, then the addition would be on the call (and the subtraction from the put, clearly). In fact, the second trader suggests 11.85% implied volatility for the USD put and 10.15% for the USD call and from this, we can infer that the ATM volatility is dealing in the market at a mid price of 11.00%.

The **VWB** is the other notable structure: it is built up by selling an ATM **STD**L and buying a symmetric Delta strangle, if one wishes to be long the **VWB**. On the contrary, by buying the straddle and selling the strangle, one is short the **VWB**. The strangle is just the sum of a (base currency) call and put both struck at a level yielding the specified level of Delta (without any consideration of its sign). The 25% Delta is the most traded **VWB**.

Since the structure, as already mentioned, has to be Vega-weighted and since the Vega of the straddle is greater the Vega of the strangle, the quantity of the former has to be smaller than the quantity of the latter. Indicating as **vwb** the butterfly’s price in volatility terms, at time t we can write the price of a 25% Delta **VWB** expiring in T as:

$$\mathbf{vwb}(t, T; 25) = 0.5(\sigma_{25c}(t, T) + \sigma_{25p}(t, T)) - \sigma_{ATM}(t, T) \quad (1.3)$$

This is how quotations for **VWB** appear in the interbank market.

The amount of the **VWB** is, as usual, expressed in terms of base currency units and referred to the amount of the ATM **STD**L (with the same convention as above) that is traded against the Vega-weighted amount of the strangle (whose total is evenly split between the 25 Delta call and the 25 Delta put).

Example 1.4.3. Hereafter a conversation is shown between two traders to deal a 25 Delta **VWB**:

> Pls, EURJPY 1Y 25D fly in 250.
> 0.275 0.375
> 0.375 pls, spot ref 158.25.
> OK, vol for atm 10.90

The first line is the request for a quote for a EURJPY 25 Delta **VWB** (“fly” is the shorthand used for it in conversations) in an amount of 250 million EUR for the ATM **STD**L. The quote is in the second line and it is the amount that has to be added to the ATM volatility to get the implied volatility for the 25D EUR call and EUR put. The first trader buys the **VWB** by applying the offer at 0.375% and suggesting the reference for the FX spot rate EURJPY. So they will buy the strangle and sell the straddle. The second trader indicates the implied volatility they will use to calculate the premium for the ATM **STD**L and the 25 Delta strangle (with an implied volatility set equal to $10.90\% + 0.375\% = 11.275\%$). The strikes will be determined by means of the FX spot rate used as reference and the implied volatilities above. The amount on the strangle will be calculated so that the total Vega of the structure is nil. Assume that 1.5 times the ATM **STD**L amount is needed for the strangle, hence the first trader buys $375/2 = 187.5$ million EUR per leg on the 25 Delta strangle and sells $250/2 = 125$ million EUR per leg in the ATM **STD**L.

We will later examine in more detail the **RR** and **VWB**: they deserve special attention since they allow us, together with the ATM **STD**L, to take exposures to the shape of the volatility matrix.

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