

Chapter 1

Introduction

All models are wrong, but some models are useful

We present in this book a financially motivated extension of the LIBOR market model that reproduces for all strikes and maturities the prices of the plain-vanilla hedging instruments (swaptions and caplets) produced by the SABR model. In other words, our extension of the LIBOR market model accurately recovers in a financially motivated manner the whole of the SABR smile surface.

As the SABR model has become the ‘market standard’ for European options, just the recovery of the smile surface by a dynamic model could be regarded as a useful achievement in itself. However, we have tried to do more. As we have stressed in the opening sentences, we have tried to accomplish this task in a way that we consider financially justifiable.

Our reason for insisting on financial reasonableness is not (just) an aesthetic one. We believe that the quality of a derivatives model should be judged not just on the basis of its ability to price today’s hedging instruments, but also on the basis of the quality of the hedges it suggests. We believe that these hedges can be good only if the model is rooted in empirical financial reality. The ‘empirical financial reality’ of relevance for the pricing and hedging of complex derivatives is the dynamics of the smile surface. We explain below why we believe that this is the case.

We are therefore not just offering yet another model. We present a ‘philosophy’ of option pricing that takes into account the realities of the industry needs (e.g., the need to calibrate as accurately as possible to the plain-vanilla reference hedging instruments, the need to obtain prices and hedges in reasonable time) while reproducing a realistic future evolution of the smile surface (our ‘financial reality’).

Until recently choosing between fitting today’s prices very accurately and being respectful of ‘financial reality’ (given our meaning of the term) entailed making hard choices. For instance, some approaches, such as local-volatility modelling (see, e.g., Dupire (1994), Derman and Kani (1994)), fulfilled (by construction) very well the first set of requirements (perfect fitting of today’s smile). This made local volatility models very popular with some traders. Yet, the dynamics of the smile these models implied were completely wrong. Indeed, the SABR model, which constitutes the starting point for our extension, was introduced to remedy the wrong *dynamics* imposed by the local-volatility framework.

On the other hand, financially much more palatable models, such as the Variance Gamma model (see, e.g., Madan and Seneta (1990)) and its ‘stochastic volatility’ extensions (see, e.g., Madan and Carr (1998)), have failed to gain acceptance in the trading rooms because of their computational cost and, above all, the difficulties in achieving a quick and stable calibration to current market prices. These prices may be ‘wrong’ and the Variance Gamma models ‘right’, but this is not a discussion the complex derivatives trader is interested in entering into – and probably wisely so.

We believe that these hard choices no longer have to be made. The framework we present recovers almost exactly today’s market prices of plain-vanilla options, and at the same time implies a reasonable future evolution for the smile surface. We say ‘reasonable’ and not ‘optimal’. The evolution our model implies is not the ‘best’ from an econometric point of view. Two of us (RR and RW), for instance, believe that a two-state Markov-chain model for the instantaneous volatility does a much better job at describing how smile surfaces evolve, especially in times of market turmoil. We have published extensively in this area (see, e.g., Rebonato and Kainth (2004) and White and Rebonato (2008)), and our ideas have been well received in academic circles. Yet we are aware that the approach, even after all the numerical tricks we have discovered, remains too awkward for daily use on the trading floor. It is destined to remain ‘another interesting model’. This is where the need for realism comes into play. We believe that the extension of the LMM that we present provides a plausible description of our financial reality while retaining tractability, computational speed and ease of calibration.

As we said, we take the SABR model (Hagan *et al.*) as the starting point for our extension of the LMM. This is not just because the SABR model has become the market standard to reproduce the price of European options. It is also because it is a good model for European options. Again, pragmatism certainly played a part in its specification as well. A log-normal choice for the volatility process is not ideal, both from a theoretical and (sometimes) from a computational point of view. However, the great advantages afforded by the ability to have an analytic approximation to the true prices, the ease of calibration and the stability of the fitted parameters have more than offset these drawbacks. The main strength of the SABR model, however, is that it is financially justifiable, not just a fitting exercise: the dynamics it implies for the smile evolution when the underlying changes are fundamentally correct – unlike the dynamics suggested by the even-better-fitting local-volatility model.

If the SABR model is so good, why do we need to tinker with it? The problem with the SABR model is that it treats each European option (caplet, swaption) in isolation – in its own measure. The processes for the various underlyings (the forward rates and swap rates) do not ‘talk to each other’. It is not obvious how to link these processes together in a coherent dynamics for the whole yield curve. The situation is strongly reminiscent of the pre-LMM days. In those days market practitioners were using the Black (1976) formula for different caplets and swaptions (each with its own ‘implied volatility’), but did not know how to link the processes together for the various forward rates to a coherent, arbitrage-free evolution for the whole yield curve. This is what the LMM achieved: it brought all the forward rates under a single measure, and specified dynamics that, thanks to the no-arbitrage ‘drift adjustments’, were simultaneously valid for all the underlyings. Complex instruments could then be priced (with a deterministic volatility).

We are trying to do something very similar. With our model we bring the dynamics of the various forward rates and *stochastic volatilities* under a single measure. To ensure absence of arbitrage we also derive ‘drift adjustments’. Not surprisingly, these have to be applied

both to the forward rates and to their volatilities. When this is done, complex derivatives, which depend on the joint realization of all the relevant forward rates, can now be priced.

All of this is not without a price: when the volatilities become stochastic, there is a whole new set of functions to specify (the volatilities of the volatilities). There is also a whole correlation structure to assign: forward-rate/forward-rate correlations, as in the LMM; but also the forward-rate/volatility and volatility/volatility correlations. For, say, a 10-year, quarterly deal, this could provide a fitting junky with hundreds of parameters to play with. Since implying process parameters from market prices is an inverse problem (which also has to rely on the informational efficiency of the market),¹ we are very wary of this approach. Instead, our philosophy can instead be summarized with the sound bite:

Imply from market prices what you can (really) hedge, and estimate econometrically what you cannot.

This is for us so important that we must explain what we mean. Ultimately, it goes back to our desire to reproduce the dynamics of the smile surface as well as we (realistically) can.

One may say: 'If the price of an option is equal to the cost of the instruments required for hedging, and if a model, like the local volatility one, reproduces the prices of all of today's hedging options perfectly, what else should a trader worry about?' We agree with the first part of the statement ('the price of an option is equal to the cost of the instruments required for hedging'), but the bit about 'the cost of the instruments required for hedging' refers not just to today's hedging, but *to all the hedging costs incurred throughout the life of the complex deal*. This, after all, is what pricing by dynamic replication is all about. Since volatility (vega) hedging is essential in complex derivatives trading, future re-hedging costs mean future prices of plain-vanilla options (future caplets and swaptions). Future prices of caplets and swaptions means future implied volatilities. Future implied volatilities means future smiles. This is why a plausible evolution of the smile is essential to complex derivatives pricing: it determines the future re-hedging costs that, according to the model, will be incurred during the life of the deal. If a model implies an implausible level or shape for the future smile (as local-volatility models do), it also implies implausible future prices for caplets and swaptions and therefore implausible re-hedging costs.

One of us (RR) has discussed all of this at some length in a recent book (see Rebonato (2004a), Chapter 1 in particular). Since we want to keep this book as concise and to-the-point as possible, we shall not repeat the argument in detail – matters, indeed, are a bit more complex because in a diffusive setting the theoretical status of vega hedging is at the very least dubious. Even here, however, we must say that our argument, despite its plausibility, does not enjoy universal acceptance. There is a school of thought that believes in what we call a 'fully implied' approach. In a nutshell, this approach says something like: 'Fit *all* the plain-vanilla option prices today with your model, without worrying too much whether your chosen model may imply implausible dynamics for the smile; use *all* the plain-vanilla instruments you have fitted to for your hedging; future re-hedging costs may indeed be different from what your model believes; but you will make compensating errors in your complex instrument *and in the hedges*.'

¹See the discussion in Rebonato (2006).

Again, one of us (RR) has argued at length against this view. In brief, the objections are that for the ‘all-implied’ approach to work option markets must either be perfectly informationally efficient or complete. The first requirement is appealing because it suggests that traders can be spared the hard task of carrying out complicated and painstaking econometric analyses, because the market has already done all this work for them: the information, according to this view, is already all in the prices, and we only have to extract it. While this optimistic view about the informational efficiency of the market may hold in the aggregate about very large, liquid and heavily scrutinized markets (such as the equity or bond markets), it is not obvious that it should be true in every corner of the investment landscape. In particular, it appears to me a bit too good to be true in the complex derivatives arena, as it implies, among other things, that supply and demand cannot affect the level of option prices – and hence of implied volatilities (an ‘excess’ supply of volatility by, say, investors should have no effect on the clearing levels of implied volatilities because, if it made options too ‘cheap’, it would entice pseudo-arbitrageurs to come in and restore price to fundamentals). Again, see the discussion by Rebonato (2004a) about this point.

The second line of defence for the ‘all-implied’ approach is somewhat less ambitious. It simply implies that ‘wrong’ prices can be ‘locked in’ by riskless trades – much as one can lock in a forward rate if one can trade in the underlying discount bonds: if one can trade in discount bonds of, say, six and nine months, one can lock in the future borrowing/lending rate without worrying whether this implied level is statistically plausible or not. This view, however, implies that the market in complex derivatives is complete, i.e., that one can notionally trade, or synthetically construct, a security with a unit payment in every single state of the world of relevance for the payoff of the complex security we want to price. But plain-vanilla instruments (caplets and European swaptions) emphatically do *not* span all the states of the world that affect the value of realistically complex derivatives products. The relevant question is therefore how much is left out by the completeness assumption. We believe that the answer is ‘far too much’.

Our approach therefore is to calibrate our model as accurately as possible to those instruments we are really going to use in our hedging (this is the ‘hedge what we really can’ part of our sound bite). We then try to ‘guesstimate’ as accurately as possible using econometric analysis the remaining relevant features of the future smile (remember, this ultimately means ‘of the future re-hedging costs’) and to ensure that our calibrated model reflects the gross features of these empirical findings in the whole if not in the detail. This is why we give such great importance to the econometric estimation of the dynamic variables of our models as to devote a whole part of the book (Part III) to the topic.

But, if the future smile is unknown today, what hopes can we have of calibrating our model appropriately, and therefore of guessing correctly the future re-hedging costs? Our hopes lie in the fact that the future smile surface may well be stochastic, but certain regularities *are* readily identifiable. We may not be able to guess exactly which shape the smile surface will assume in the future, but we should make sure that these identifiable regularities are broadly recovered. An informed guess, we believe, is way better than nothing. If the goal seems too modest, let us not forget that the local-volatility model miserably fails even this entry-level test of statistical acceptability.

So, we do not peddle the snake-oil of the ‘perfect model with the perfect hedge’. After all, if a substantial degree of uncertainty did not remain even after the best model was used, it would be difficult to explain why, in a competitive market, the margins enjoyed by complex derivatives traders are still so much wider than the wafer-thin margins available

in less uncertain, or more readily hedgeable, asset classes. The name of the game therefore is not to hope that we can eliminate all uncertainty (perhaps by deluding ourselves that we can 'lock in' all the current market prices). A more realistic goal for a good model is to offer the ability to reduce the uncertainty to an acceptable minimum by making as judicious a use as possible of the econometric information available.

This is what we believe our modelling approach can offer. And this is why our book is different from most other books on derivatives pricing, which tend to be heavy on stochastic calculus but worryingly thin on empirical analysis.

Finally, we are well aware that there are conditions of market stress that our model 'does not know about'. We therefore propose in the last chapter of our book a pragmatic hedging approach, inspired by the work two of us (RR and RW) have done with the two-state Markov-chain approach mentioned above. This approach can ensure a reasonable hedging strategy even in those situations when the (essentially diffusive) assumptions of our model fail miserably. This will be an unashamedly 'outside-the-model' hedging methodology, whose strength relies on two essential components: the empirical regularities of the dynamics of the smile surface; and the robustness of the fits we propose. As these are two cornerstones of our approach, we believe that we have a chance of succeeding.

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