

Chapter 1

The Quantitative Finance Timeline



2 Frequently Asked Questions in Quantitative Finance

There follows a speedy, roller-coaster of a ride through the official history of quantitative finance, passing through both the highs and lows. Where possible I give dates, name names and refer to the original sources.¹

1827 Brown The Scottish botanist, Robert Brown, gave his name to the random motion of small particles in a liquid. This idea of the random walk has permeated many scientific fields and is commonly used as the model mechanism behind a variety of unpredictable continuous-time processes. The lognormal random walk based on Brownian motion is the classical paradigm for the stock market. See Brown (1827).

1900 Bachelier Louis Bachelier was the first to quantify the concept of Brownian motion. He developed a mathematical theory for random walks, a theory rediscovered later by Einstein. He proposed a model for equity prices, a simple normal distribution, and built on it a model for pricing the almost unheard of options. His model contained many of the seeds for later work but lay 'dormant' for many years. It is told that his thesis was not a great success and, naturally, Bachelier's work was not appreciated in his lifetime. See Bachelier (1995).

1905 Einstein Albert Einstein proposed a scientific foundation for Brownian motion in 1905. He did some other clever stuff as well. See Stachel (1990).

1911 Richardson Most option models result in diffusion-type equations. And often these have to be solved numerically. The two main ways of doing this are Monte Carlo and finite differences (a sophisticated version of the binomial model).

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The very first use of the finite-difference method, in which a differential equation is discretized into a difference equation, was by Lewis Fry Richardson in 1911, and used to solve the diffusion equation associated with weather forecasting. See Richardson (1922). Richardson later worked on the mathematics for the causes of war. During his work on the relationship between the probability of war and the length of common borders between countries he stumbled upon the concept of fractals, observing that the length of borders depended on the length of the ‘ruler.’ The fractal nature of turbulence was summed up in his poem “Big whorls have little whorls that feed on their velocity, and little whorls have smaller whorls and so on to viscosity.”

1923 Wiener Norbert Wiener developed a rigorous theory for Brownian motion, the mathematics of which was to become a necessary modelling device for quantitative finance decades later. The starting point for almost all financial models, the first equation written down in most technical papers, includes the Wiener process as the representation for randomness in asset prices. See Wiener (1923).

1950s Samuelson The 1970 Nobel Laureate in Economics, Paul Samuelson, was responsible for setting the tone for subsequent generations of economists. Samuelson ‘mathematized’ both macro and micro economics. He rediscovered Bachelier’s thesis and laid the foundations for later option pricing theories. His approach to derivative pricing was via expectations, real as opposed to the much later risk-neutral ones. See Samuelson (1955).

1951 Itô Where would we be without stochastic or Itô calculus? (Some people even think finance is *only* about Itô calculus.) Kiyosi Itô showed the relationship between a stochastic differential equation for some independent variable and the stochastic differential equation for a function of that variable. One of the starting points for classical derivatives theory is

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the lognormal stochastic differential equation for the evolution of an asset. Itô's lemma tells us the stochastic differential equation for the value of an option on that asset.

In mathematical terms, if we have a Wiener process X with increments dX that are normally distributed with mean zero and variance dt , then the increment of a function $F(X)$ is given by

$$dF = \frac{dF}{dX}dX + \frac{1}{2} \frac{d^2F}{dX^2}dt$$

This is a very loose definition of Itô's lemma but will suffice. See Itô (1951).

1952 Markowitz Harry Markowitz was the first to propose a modern quantitative methodology for portfolio selection. This required knowledge of assets' volatilities and the correlation between assets. The idea was extremely elegant, resulting in novel ideas such as 'efficiency' and 'market portfolios.' In this Modern Portfolio Theory, Markowitz showed that combinations of assets could have better properties than any individual assets. What did 'better' mean? Markowitz quantified a portfolio's possible future performance in terms of its expected return and its standard deviation. The latter was to be interpreted as its risk. He showed how to optimize a portfolio to give the maximum expected return for a given level of risk. Such a portfolio was said to be 'efficient.' The work later won Markowitz a Nobel Prize for Economics but is problematic to use in practice because of the difficulty in measuring the parameters 'volatility,' and, especially, 'correlation,' and their instability.

1963 Sharpe, Lintner and Mossin William Sharpe of Stanford, John Lintner of Harvard and Norwegian economist Jan Mossin independently developed a simple model for pricing risky assets. This Capital Asset Pricing Model (CAPM) also reduced the number of parameters needed for portfolio selection from those needed by Markowitz's Modern Portfolio Theory,

making asset allocation theory more practical. See Sharpe, Alexander and Bailey (1999), Lintner (1965) and Mossin (1966).

1966 Fama Eugene Fama concluded that stock prices were unpredictable and coined the phrase ‘market efficiency.’ Although there are various forms of market efficiency, in a nutshell the idea is that stock market prices reflect all publicly available information, and that no person can gain an edge over another by fair means. See Fama (1966).

1960s Sobol’, Faure, Hammersley, Haselgrove and Halton... Many people were associated with the definition and development of quasi random number theory or low-discrepancy sequence theory. The subject concerns the distribution of points in an arbitrary number of dimensions in order to cover the space as efficiently as possible, with as few points as possible (see Figure 1.1). The methodology is used in the evaluation of multiple integrals among other things. These ideas would find a use in finance almost three decades later. See Sobol’ (1967), Faure (1969), Hammersley & Handscomb (1964), Haselgrove (1961) and Halton (1960).

1968 Thorp Ed Thorp’s first claim to fame was that he figured out how to win at casino Blackjack, ideas that were put into practice by Thorp himself and written about in his best-selling *Beat the Dealer*, the “book that made Las Vegas change its rules.” His second claim to fame is that he invented and built, with Claude Shannon, the information theorist, the world’s first wearable computer. His third claim to fame is that he used the ‘correct’ formulæ for pricing options, formulæ that were rediscovered and originally published several years later by the next three people on our list. Thorp used these formulæ to make a fortune for himself and his clients in the first ever quantitative finance-based hedge fund. He proposed dynamic hedging as a way of

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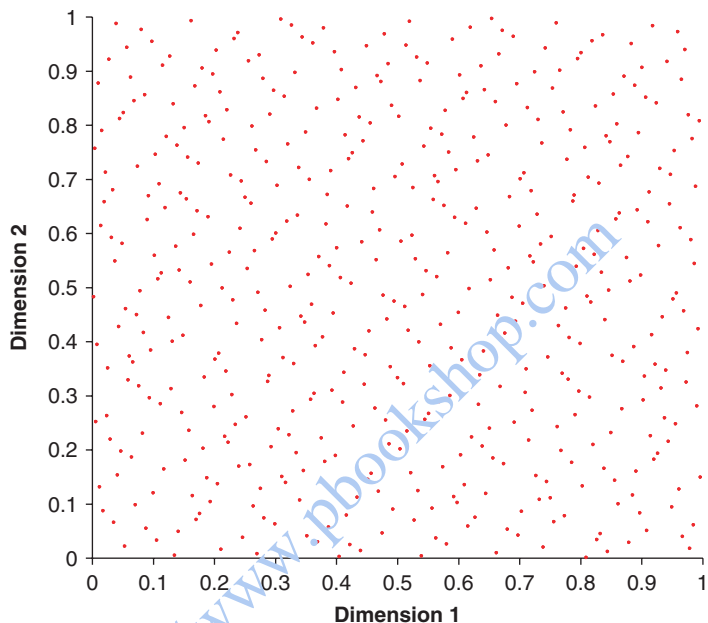


Figure 1.1: They may not look like it, but these dots are distributed deterministically so as to have very useful properties.

removing more risk than static hedging. See Thorp (2002) for the story behind the discovery of the Black–Scholes formulæ.

1973 Black, Scholes and Merton Fischer Black, Myron Scholes and Robert Merton derived the Black–Scholes equation for options in the early seventies, publishing it in two separate papers in 1973 (Black & Scholes, 1973, and Merton, 1973). The date corresponded almost exactly with the trading of call options on the Chicago Board Options Exchange. Scholes and Merton won the Nobel Prize for Economics in 1997. Black had died in 1995.

The Black–Scholes model is based on geometric Brownian motion for the asset price S

$$dS = \mu S dt + \sigma S dX.$$

The Black–Scholes partial differential equation for the value V of an option is then

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

1974 Merton, again In 1974 Robert Merton (Merton, 1974) introduced the idea of modelling the value of a company as a call option on its assets, with the company's debt being related to the strike price and the maturity of the debt being the option's expiration. Thus was born the structural approach to modeling risk of default, for if the option expired out of the money (i.e. assets had less value than the debt at maturity) then the firm would have to go bankrupt.

Credit risk became big, huge, in the 1990s. Theory and practice progressed at rapid speed during this period, urged on by some significant credit-led events, such as the Long Term Capital Management mess. One of the principals of LTCM was Merton who had worked on credit risk two decades earlier. Now the subject really took off, not just along the lines proposed by Merton but also using the Poisson process as the model for the random arrival of an event, such as bankruptcy or default. For a list of key research in this area see Schönbucher (2003).

1977 Boyle Phelim Boyle related the pricing of options to the simulation of random asset paths (Figure 1.2). He showed how to find the fair value of an option by generating lots of possible future paths for an asset and then looking at the average that the option had paid off. The future important

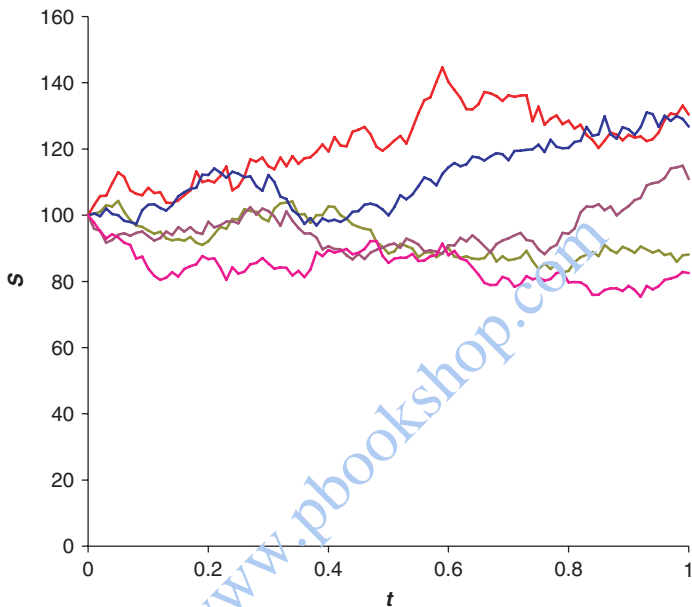


Figure 1.2: Simulations like this can be easily used to value derivatives.

role of Monte Carlo simulations in finance was assured. See Boyle (1977).

1977 Vasicek So far quantitative finance hadn't had much to say about pricing interest rate products. Some people were using equity option formulæ for pricing interest rate options, but a consistent framework for interest rates had not been developed. This was addressed by Vasicek. He started by modelling a short-term interest rate as a random walk and concluded that interest rate derivatives could be valued using equations similar to the Black–Scholes partial differential equation.

Oldrich Vasicek represented the short-term interest rate by a stochastic differential equation of the form

$$dr = \mu(r, t) dt + \sigma(r, t) dX.$$

The bond pricing equation is a parabolic partial differential equation, similar to the Black–Scholes equation. See Vasicek (1977).

1979 Cox, Ross and Rubinstein Boyle had shown how to price options via simulations, an important and intuitively reasonable idea, but it was these three, John Cox, Stephen Ross and Mark Rubinstein, who gave option-pricing capability to the masses.

The Black–Scholes equation was derived using stochastic calculus and resulted in a partial differential equation. This was not likely to endear it to the thousands of students interested in a career in finance. At that time these were typically MBA students, not the mathematicians and physicists that are nowadays found on Wall Street. How could MBAs cope? An MBA was a necessary requirement for a prestigious career in finance, but an ability to count beans is not the same as an ability to understand mathematics. Fortunately Cox, Ross and Rubinstein were able to distil the fundamental concepts of option pricing into a simple algorithm requiring only addition, subtraction, multiplication and (twice) division. Even MBAs could now join in the fun. See Cox, Ross & Rubinstein (1979) and Figure 1.3.

1979–81 Harrison, Kreps and Pliska Until these three came onto the scene quantitative finance was the domain of either economists or applied mathematicians. Mike Harrison and David Kreps, in 1979, showed the relationship between option prices and advanced probability theory, originally in discrete time. Harrison and Stan Pliska in 1981 used the same ideas but in continuous time. From that moment until the mid 1990s applied mathematicians hardly got a look in. Theorem,

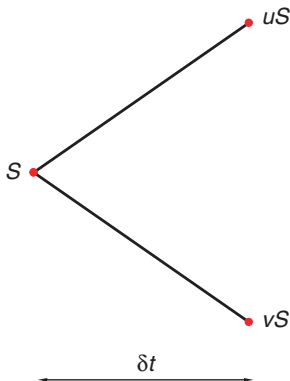


Figure 1.3: The branching structure of the binomial model.

proof everywhere you looked. See Harrison & Kreps (1979) and Harrison & Pliska (1981).

1986 Ho and Lee One of the problems with the Vasicek framework for interest-rate derivative products was that it didn't give very good prices for bonds, the simplest of fixed-income products. If the model couldn't even get bond prices right, how could it hope to correctly value bond options? Thomas Ho and Sang-Bin Lee found a way around this, introducing the idea of yield-curve fitting or calibration. See Ho & Lee (1986).

1992 Heath, Jarrow and Morton Although Ho and Lee showed how to match theoretical and market prices for simple bonds, the methodology was rather cumbersome and not easily generalized. David Heath, Robert Jarrow and Andrew Morton (HJM) took a different approach. Instead of modelling just a short rate and deducing the whole yield curve, they modelled the random evolution of the whole yield curve. The initial yield curve, and hence the value of simple interest

rate instruments, was an input to the model. The model cannot easily be expressed in differential equation terms and so relies on either Monte Carlo simulation or tree building. The work was well known via a working paper, but was finally published, and therefore made respectable in Heath, Jarrow & Morton (1992).

1990s Cheyette, Barrett, Moore and Wilmott When there are many underlyings, all following lognormal random walks, you can write down the value of any European non-path-dependent option as a multiple integral, one dimension for each asset. Valuing such options then becomes equivalent to calculating an integral. The usual methods for quadrature are very inefficient in high dimensions, but simulations can prove quite effective. Monte Carlo evaluation of integrals is based on the idea that an integral is just an average multiplied by a 'volume.' And since one way of estimating an average is by picking numbers at random, we can value a multiple integral by picking integrand values at random and summing. With N function evaluations, taking a time of $O(N)$ you can expect an accuracy of $O(1/N^{1/2})$, independent of the number of dimensions. As mentioned above, breakthroughs in the 1960s on low-discrepancy sequences showed how clever, non-random, distributions could be used for an accuracy of $O(1/N)$, to leading order. (There is a weak dependence on the dimension.) In the early 1990s several groups of people were simultaneously working on valuation of multi-asset options. Their work was less of a breakthrough than a transfer of technology.

They used ideas from the field of number theory and applied them to finance. Nowadays, these low-discrepancy sequences are commonly used for option valuation whenever random numbers are needed. A few years after these researchers made their work public, a completely unrelated group at Columbia University successfully patented the work. See Oren Cheyette (1990) and John Barrett, Gerald Moore & Paul Wilmott (1992).

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1994 Dupire, Rubinstein, Derman and Kani Another discovery was made independently and simultaneously by three groups of researchers in the subject of option pricing with deterministic volatility. One of the perceived problems with classical option pricing is that the assumption of constant volatility is inconsistent with market prices of exchange-traded instruments. A model is needed that can correctly price vanilla contracts, and then price exotic contracts consistently. The new methodology, which quickly became standard market practice, was to find the volatility as a function of underlying and time that when put into the Black–Scholes equation and solved, usually numerically, gave resulting option prices which matched market prices. This is what is known as an inverse problem: use the ‘answer’ to find the coefficients in the governing equation. On the plus side, this is not too difficult to do in theory. On the minus side, the practice is much harder, the sought volatility function depending very sensitively on the initial data. From a scientific point of view there is much to be said against the methodology. The resulting volatility structure never matches actual volatility, and even if exotics are priced consistently it is not clear how to best hedge exotics with vanillas in order to minimize any model error. Such concerns seem to carry little weight, since the method is so ubiquitous. As so often happens in finance, once a technique becomes popular it is hard to go against the majority. There is job safety in numbers. See Emanuel Derman & Iraj Kani (1994), Bruno Dupire (1994) and Mark Rubinstein (1994).

1996 Avellaneda and Parás Marco Avellaneda and Antonio Parás were, together with Arnon Levy and Terry Lyons, the creators of the uncertain volatility model for option pricing. It was a great breakthrough for the rigorous, scientific side of finance theory, but the best was yet to come. This model, and many that succeeded it, was nonlinear. Nonlinearity in an option pricing model means that the value of a portfolio of contracts is not necessarily the same as the sum of the values of its constituent parts. An option will have a different

value depending on what else is in the portfolio with it, and an exotic will have a different value depending on what it is statically hedged with. Avellaneda and Parás defined an exotic option's value as the highest possible marginal value for that contract when hedged with any or all available exchange-traded contracts. The result was that the method of option pricing also came with its own technique for static hedging with other options. Prior to their work the only result of an option pricing model was its value and its delta, only dynamic hedging was theoretically necessary. With this new concept, theory became a major step closer to practice. Another result of this technique was that the theoretical price of an exchange-traded option exactly matched its market price. The convoluted calibration of volatility surface models was redundant. See Avellaneda & Parás (1996).

1997 Brace, Gatarek and Musiela Although the HJM interest rate model had addressed the main problem with stochastic spot rate models, and others of that ilk, it still had two major drawbacks. It required the existence of a spot rate and it assumed a continuous distribution of forward rates. Alan Brace, Dariusz Gatarek & Marek Musiela (1997) got around both of those difficulties by introducing a model which only relied on a discrete set of rates – ones that actually are traded. As with the HJM model the initial data are the forward rates so that bond prices are calibrated automatically. One specifies a number of random factors, their volatilities and correlations between them, and the requirement of no arbitrage then determines the risk-neutral drifts. Although B, G and M have their names associated with this idea many others worked on it simultaneously.

2000 Li As already mentioned, the 1990s saw an explosion in the number of credit instruments available, and also in the growth of derivatives with multiple underlyings. It's not a great step to imagine contracts depending on the default of many underlyings. Examples of these are the once ubiquitous

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Collateralized Debt Obligations (CDOs). But to price such complicated instruments requires a model for the interaction of many companies during the process of default. A probabilistic approach based on copulas was proposed by David Li (2000). The copula approach allows one to join together (hence the word ‘copula’) default models for individual companies in isolation to make a model for the probabilities of their joint default. The idea was adopted universally as a practical solution to a complicated problem. However, with the recent financial crisis the concept has come in for a lot of criticism.

2002 Hagan, Kumar, Lesniewski and Woodward There has always been a need for models that are both fast and match traded prices well. The interest-rate model of Pat Hagan, Deep Kumar, Andrew Lesniewski and Diana Woodward (2002), which has come to be called the SABR (stochastic, α , β , ρ) model, is a model for a forward rate and its volatility, both of which are stochastic. This model is made tractable by exploiting an asymptotic approximation to the governing equation that is highly accurate in practice. The asymptotic analysis simplifies a problem that would otherwise have to be solved numerically. Although asymptotic analysis has been used in financial problems before, for example in modelling transaction costs, this was the first time it really entered mainstream quantitative finance.

August 2007 quantitative finance in disrepute In early August 2007 several hedge funds using quantitative strategies experienced losses on such a scale as to bring the field of quantitative finance into disrepute. From then, and through 2008, trading of complex derivative products in obscene amounts using simplistic mathematical models almost brought the global financial market to its knees: Lend to the less-than-totally-creditworthy for home purchase, repackage these mortgages for selling on from one bank to another, at each stage adding complexity, combine with overoptimistic

ratings given to these products by the ratings agencies, with a dash of moral hazard thrown in, base it all on a crunchy base of a morally corrupt compensation scheme, and you have the recipe for the biggest financial collapse in decades. Out of this many people became very, very rich, while in many cases the man in the street lost his life savings. And financial modelling is what made this seem all so simple and safe.

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And Now a Brief Unofficial History!

Espen Gaarder Haug, as well as being an option trader, author, lecturer, researcher, gardener, soldier, and collector of option-pricing formulæ, is also a historian of derivatives theory. In his excellent book *Derivatives: Model on Models* (John Wiley and Sons Ltd, 2007) he gives the ‘alternative’ history of derivatives, a history often ignored for various reasons. He also keeps us updated on his findings via his blog <http://www.wilmott.com/blogs/collector>. Here are a few of the many interesting facts Espen has unearthed.

1688 *de la Vega* Possibly a reference to put–call parity. But then possibly not. De la Vega’s language is not particularly precise.

1900s *Higgins and Nelson* They appear to have some grasp of delta hedging and put–call parity.

1908 *Bronzin* Publishes a book that includes option formulæ, and seems to be using risk neutrality. But the work is rapidly forgotten!

1915 Mitchell, 1926 Oliver and 1927 Mills They all described the high-peak/fat-tails in empirical price data.

1956 Kruizenga and 1961 Reinach They definitely describe put-call parity. Reinach explains 'conversion,' which is what we know as put-call parity, he also understands that it does not necessarily apply for American options.

1962 Mandelbrot In this year Benoit Mandelbrot wrote his famous paper on the distribution of cotton price returns, observing their fat tails.

1970 Arnold Bernhard & Co They describe market-neutral delta hedging of convertible bonds and warrants. And show how to numerically find an approximation to the delta.

For more details about the underground history of derivatives see Espen's excellent book (2007).

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