CHAPTER 1

Cost of Capital Basics

INTRODUCTION

The cost of capital is the expected rate of return that market participants require in order to attract funds to a particular investment. In economic terms, the cost of capital for a particular investment is an opportunity cost—the cost of forgoing the next best alternative investment. In this sense, it relates to the economic principle of substitution, that is, an investor will not invest in a particular asset if there is a more attractive substitute.

The term *market* refers to the universe of investors who are reasonable candidates to fund a particular investment. Capital or funds are usually provided in the form of cash, although in some instances capital may be provided in the form of other assets. The cost of capital usually is expressed in percentage terms, that is, the annual amount of dollars that the investor requires or expects to realize, expressed as a percentage of the dollar amount invested.

Put another way:

> Since the cost of anything can be defined as the price one must pay to get it, the cost of capital is the return a company must promise in order to get capital from the market, either debt or equity. A company does not set its own cost of capital; it must go into the market to discover it. Yet meeting this cost is the financial market's one basic yardstick for determining whether a company's performance is adequate.[1]

As the quote suggests, most of the information for estimating the cost of capital for a business, security, or project comes from the investment market. The cost of capital is always an *expected* (or forward-looking) return. Thus, analysts and would-be investors never actually observe the market's views as to expected returns at the time of their investment. However, we often form our views of the future by analyzing historical market data.

COMPONENTS OF A CAPITAL STRUCTURE

The term *capital* in this context means the components of an entity's capital structure. The primary components of a capital structure include:

- Debt capital
- Preferred equity capital (i.e., stock, partnership, limited liability company, or other type of entity interests with preference features, such as seniority in receipt of dividends or liquidation proceeds)
- Common equity capital (i.e., stock, partnership, limited liability company, or other type of entity interests at the lowest or residual level of the capital structure)

There may be more than one subcategory in any or all of the listed categories of capital. Also, there may be related forms of capital, such as warrants or options. Each component of an entity's capital structure has its own unique cost, depending primarily on its respective risk.

Simply and cogently stated, "The cost of equity is the rate of return investors require on an equity investment in a firm."[2]

[1] Mike Kaufman, "Profitability and the Cost of Capital," in *Handbook of Budgeting*, 4th ed., Robert Rachlin, ed. (New York: John Wiley & Sons, 1999), 8–3.
[2] Aswath Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, 2nd ed. (Hoboken, NJ: John Wiley & Sons, 2002), 182.

When we talk about the cost of ownership capital (i.e., the expected return to an equity investor), we usually use the phrase *cost of equity capital*. When we talk about the cost of capital to the business overall (i.e., the average cost of capital for both equity ownership interests and debt interests), we commonly use the phrases *weighted average cost of capital* (WACC) or *blended cost of capital* or *overall cost of capital*.

CHARACTERISTICS OF COST OF CAPITAL

Cost of Capital Is Forward-Looking

The cost of capital represents investors' *expectations*. There are three elements to these expectations:

1. The risk-free rate, which includes:
   - *Rental rate*. A real return for lending the funds risk-free, thus forgoing consumption for which the funds otherwise could be used.
   - *Inflation*. The expected rate of inflation over the term of the risk-free investment.
   - *Maturity risk or investment rate risk*. The risk that the investment's principal market value will rise or fall during the period to maturity as a function of changes in the general level of interest rates.
2. Risk—the uncertainty as to when and how much cash flow or other economic income will be received.

The combination of the first two items comprising the risk-free rate is sometimes referred to as the *time value of money*. While these expectations, including assessment of risk, may be different for different investors, the market tends to form a consensus with respect to a particular investment or category of investments. That consensus determines the cost of capital for investments of varying levels of risk.

The cost of capital, derived from investors' expectations and the market's consensus of those expectations, is applied to *expected economic income, usually measured in terms of net cash flows*. We convert the stream of expected economic benefits to its present value equivalent to compare investment alternatives of similar or differing levels of risk. *Present value*, in this context, refers to the dollar amount that a rational and well-informed investor would be willing to pay today for the stream of expected economic income. In mathematical terms, the cost of capital is the percentage rate of return that equates the stream of expected economic income with its present cash value.

Cost of Capital Is Based on Market Value

The cost of capital is the expected rate of return on some base value. That base value is measured as the market value of an asset, not its book value, par value, or carrying value. For example, the yield to maturity shown in the bond quotations in the financial press is based on the closing market price of a bond, not on its face value. Similarly, the implied cost of equity for a company's stock is based on the market price per share at which it trades, not on the company's book value per share of stock. The cost of capital is estimated from market data. This data refers to expected returns relative to market prices. By applying the cost of capital derived from market expectations to the expected net cash flows (or other measure of economic income) from the investment or project under consideration, the market value can be estimated.

Cost of Capital Is Usually Stated in Nominal Terms

Keep in mind that we have talked about expectations including inflation. Assuming inflationary expectations, the return an investor requires includes compensation for reduced purchasing power of the currency over the life of the investment. Therefore, when the analyst or investor applies the cost of capital to expected returns in order to estimate value, he or she must also include expected inflation in those expected returns.

This obviously assumes that investors have reasonable consensus expectations regarding inflation. For countries subject to unpredictable hyperinflation, it is sometimes more practical to estimate the cost of capital in real terms rather than in nominal terms and apply it to expected net cash flows expressed in real terms.

COST OF CAPITAL EQUALS THE DISCOUNT RATE

The essence of the cost of capital is that it is the percentage return that equates expected economic income with present value. The expected rate of return in this context is called a *discount rate*. By discount rate, the financial community means an *annually compounded rate* at which each increment of expected economic income is discounted back to its present value. A discount rate reflects both the time value of money and risk. Therefore in its totality it represents the cost of capital. The sum of the discounted present values of each future period's net cash flow or other measure of return equals the present value of the investment, reflecting the expected amounts of return over the life of the investment. The terms *discount rate*, *cost of capital*, and *required rate of return* are often used interchangeably.

The economic income referenced here represents *total expected benefits*. In other words, this economic income includes increments of cash flow realized by the investor while holding the investment, as well as proceeds to the investor upon liquidation of the investment. The rate at which these expected future total returns are reduced to present value is the discount rate, which is the *cost of capital* (required rate of return) for a particular investment.

Cost of Capital Is the Proper Discount Rate

The cost of capital is customarily used as a discount rate to convert expected economic income to a present value. In this context, let us keep in mind the critical characteristics of a discount rate:

Definition: A *discount rate* is a yield rate used to convert anticipated future economic income (payments or receipts) into present value (i.e., a cash value as of a specified valuation date).

A discount rate represents the *total expected rate of return* that the investor requires on the amount invested.

Usually analysts and investors make the simplifying assumption that the cost of capital is constant over the life of the investment and use the same cost of capital to apply to each future period's expected economic income. There are, however, cases in which analysts might choose to estimate a discrete cost of capital to apply to the expected economic income in each future period. Examples include cases where the analyst anticipates a changing weighted average cost of capital because of a changing capital structure or cases where the risk characteristics of the economic income change (e.g., the net cash flows in the early years are "guaranteed" due to contracts with customers and are risky in later years).

Present Value Formula

The use of the cost of capital to estimate present value thus requires two sets of estimates:

1. The *numerator*. The expected amount of economic income (e.g., the net cash flow) to be received from the investment in each future period over the life of the investment.
2. The *denominator*. A function of the discount rate, which is the cost of capital, which, in turn, is the required rate of return. This function is usually written as $(1+k)^n$

where: k = discount rate
n = number of periods into the future when the returns are expected to be realized

Converting the concepts into a mathematical formula, we have the following, which is the essence of using cost of capital to estimate present value.

(Formula 1.1)

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \cdots + \frac{NCF_n}{(1+k)^n}$$

where: PV = present value
$NCF_1 \; . . . \; NCF_n$ = net cash flow (or other measure of economic income) expected in each of the periods 1 through n, n being the final cash flow in the life of the investment
k = cost of capital applicable to the defined stream of net cash flow
n = number of periods

The critical job for the analyst is to match the cost of capital estimate to the definition of the economic income stream being discounted. This is largely a function of reflecting in the cost of capital estimate the degree of risk inherent in the expected cash flows being discounted.

NET CASH FLOW

Throughout this book we usually assume that the measure of economic income to which the cost of capital will be applied is *net cash flow* (sometimes called *free cash flow*). Net cash flow represents discretionary cash available to be paid out to stakeholders of an entity (providers of capital to the entity)—for example, interest, debt payments, dividends, withdrawals—without jeopardizing the projected ongoing operations of the entity. Net cash flow to equity is that cash flow available to the equity holders, usually common equity.

The Preferred Economic Income Measure

Net cash flow is the measure of economic income on which most financial analysts today prefer to focus for both valuation and capital investment purposes. Net cash flow represents money available to stakeholders, assuming the business owned by the entity is a going concern and the entity is able to support the projected operations. Net cash flow can also be used to evaluate liquidation scenarios. Although the contemporary literature of corporate finance widely embraces a preference for net cash flow as the relevant economic income measure to which to apply cost of capital for valuation and decision making, there is still a contingent of analysts who prefer to focus on reported or adjusted accounting income.[3]

We discuss adopting the cost of capital developed for net cash flows to other measures of economic income later in this chapter.

Defining Net Cash Flow

Net cash flow is generally defined as cash that a business or project does not have to retain and reinvest in itself in order to generate the projected cash flows in future years. In other words, it is cash *available to be paid out* in any year to the owners of capital without jeopardizing the business's expected cash flow generating capability in future years.

The net cash flow is available to be distributed to the investors *or* reinvested in some incremental project not reflected in the net cash flows that have been discounted. That reinvestment results in incremental value in future years.

Net cash flow is sometimes called *free cash flow*. It is also sometimes called *net free cash flow*, although this phrase seems redundant. With finance terminology being as ambiguous as it is, minor variations in the definitions of these terms frequently arise, making it essential to clearly define the measure of income to be employed in the valuation.

[3] See, for example, Z. Christopher Mercer, *Valuing Financial Institutions* (Homewood, Ill.: Business One Irwin, 1992), Chapter 13; and his article "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates," *Business Valuation Review* (December 1989): 147–156.

Net Cash Flow to Common Equity Capital In valuing *equity capital* by discounting or capitalizing expected net cash flows (keeping in mind the important difference between discounting and capitalizing, as discussed later in the chapter, *net cash flow to equity* (NCF_e in our notation system) is defined as:

(Formula 1.2)

Net income to common equity (after income taxes)

Plus: Non-cash charges (depreciation, amortization, deferred revenues, and deferred income taxes)

Minus: Capital expenditures (amount necessary to support projected revenues and expenses)

Minus: Additions to net working capital (amount necessary to support projected revenues)

Minus: Dividends on preferred equity capital

Plus: Cash from increases in the preferred equity or debt components of the capital structure (amount necessary to support projected revenues)

Minus: Repayments of any debt components or retirement of any preferred components of the capital structure

Equals: Net cash flow to common equity capital

Capital expenditures are those amounts needed to match the revenue and expense forecasts. That is, the capital expenditures are those amounts needed for replacement of plant and/or equipment that are retired in the normal course of business, those amounts needed for increases in capacity consistent with the projected revenue (e.g., increased number of machines, increased warehouse space, etc.), and those amounts needed for replacement of existing plant and/or equipment consistent with projected expenses (e.g., replacement of inefficient equipment with more efficient equipment).

Net working capital excludes (1) any excess cash and investments that are not needed to support the level of business activity in the projected revenues and (2) any debt classified as short-term that is a component of the capital structure (e.g., the amount included in current liabilities for the current portion of long-term debt).

Because we are only including amounts of investment in net working capital and capital expenditures needed for the projected revenues and expenses included in the projected net cash flows to be discounted, we can term these *sustainable net cash flows*.

Net cash flow to equity is also called *free cash flow to equity* (FCF_e).

Net Cash Flow to Invested Capital In valuing the entire *invested capital* of a business or project by discounting or capitalizing expected cash flows, *net cash flow to invested capital* or *net cash flow to the firm* (NCF_f in our notation system) is defined as:

(Formula 1.3)

Net income to common equity (after income taxes)

Plus: Non-cash charges (depreciation, amortization, deferred revenues, and deferred income taxes)

Minus: Capital expenditures (amount necessary to support projected revenues and expenses)

Minus: Additions to net working capital (amount necessary to support projected revenues)

Plus: Interest expense (net of the tax deduction resulting from interest as a tax-deductible expense)

Plus: Dividends on preferred equity capital

Equals: Net cash flow to invested capital

The amounts of capital expenditures and additions to net working capital are consistent with the projections of revenues and expenses and the amounts previously defined (in the net cash flow to common equity capital).

In other words, NCF_f adds back interest (tax-affected because interest is a tax-deductible expense) because invested capital includes the debt on which the interest is paid. Interest is the payment to the debt component of the invested capital. It also adds back dividends on preferred stock for the same reason (i.e., invested capital includes the preferred capital on which the dividends are paid).

Net cash flow to invested capital is also called *free cash flow to the firm* (FCF_f).

Occasionally an analyst treats earnings before interest, taxes, depreciation, and amortization (EBITDA) as if it were equivalent to net cash flow to invested capital. This error may be a significant matter because the analyst has added back the non-cash charges but ignored the requisite capital expenditures and additions to net working capital necessary to sustain the business as projected.

When we discount net cash flow to equity, the appropriate discount rate is the cost of equity capital. When we discount net cash flow to all invested capital, the appropriate discount rate is the overall cost of capital or WACC.

Net Cash Flows Should Be Probability Weighted Expected Values

Net cash flows to be discounted or capitalized should be *statistical expected values*, that is, (mean) *probability-weighted* net cash flows. In the real world, it is far more common for realized net cash flows to be below forecast than above, as we explain later. A valuation that does not take this factor into account will overvalue a business.

If the distribution of possible net cash flows in amount and likelihood in each period is symmetrical above and below the most likely net cash flow in that period, then the most likely net cash flow is equal to the probability-weighted net cash flow (the mathematical expected value of the distribution). However, many times distributions of possible net cash flows for any given period are skewed. This is where probability weighting comes into play. Exhibit 1.1 shows the calculation of the probability-weighted expected values of projected net cash flows under a symmetrically distributed scenario (Scenario A) and under a skewed

EXHIBIT 1.1 Example of Net Cash Flow Expectations

Scenario A—Symmetrical Net Cash Flow Expectation

Projected Net Cash Flows	Probability of Occurrence	Probability Weighted Value
$1,600.00	0.01	$16
1,500.00	0.09	135
1,300.00	0.20	260
1,000.00	0.40	400
700.00	0.20	140
500.00	0.09	45
400.00	0.01	4
	100%	$1,000

Scenario B—Skewed Net Cash Flow Expectation

Projected Net Cash Flows	Probability of Occurrence	Probability Weighted Value
$1,600.00	0.01	$16
1,500.00	0.02	30
1,300.00	0.05	65
1,000.00	0.35	350
700.00	0.25	175
500.00	0.20	100
(100.00)	0.10	(10)
(600.00)	0.02	(12)
	100%	$714

distribution scenario (Scenario B). Exhibit 1.2 portrays the information in Exhibit 1.1 graphically.

In both scenario A and scenario B of Exhibit 1.1, the most likely net cash flow is $1,000. In scenario A, the expected value (probability weighted) is also $1,000. But in scenario B, the expected value is only $714. In scenario B, $714 is the figure that should appear in the numerator of the discounted cash flow formula, not $1,000.

Most analysts do not have the benefit of receiving, or the time or information to develop, a probability distribution for each year's expected net cash flow (and it is not a common practice to develop one). However, analysts should be aware of the concept when deciding on the amount of each expected net cash flow to be discounted.

Many analysts first think in terms of symmetrical distributions. But most businesses have a maximum capacity for producing their services or goods in any one year. For example, in scenario B, the business in any year may run up against capacity constraints to increase revenues and net cash flows (except with short spurts of multi-shift seven-day-per-week production). But on the downside, the business is more likely to lose sales and experience reduced cash flows. So in any one year, there is a great likelihood that the distribution of expected cash flows is skewed. This does not mean that the net cash flows are identical in future years.

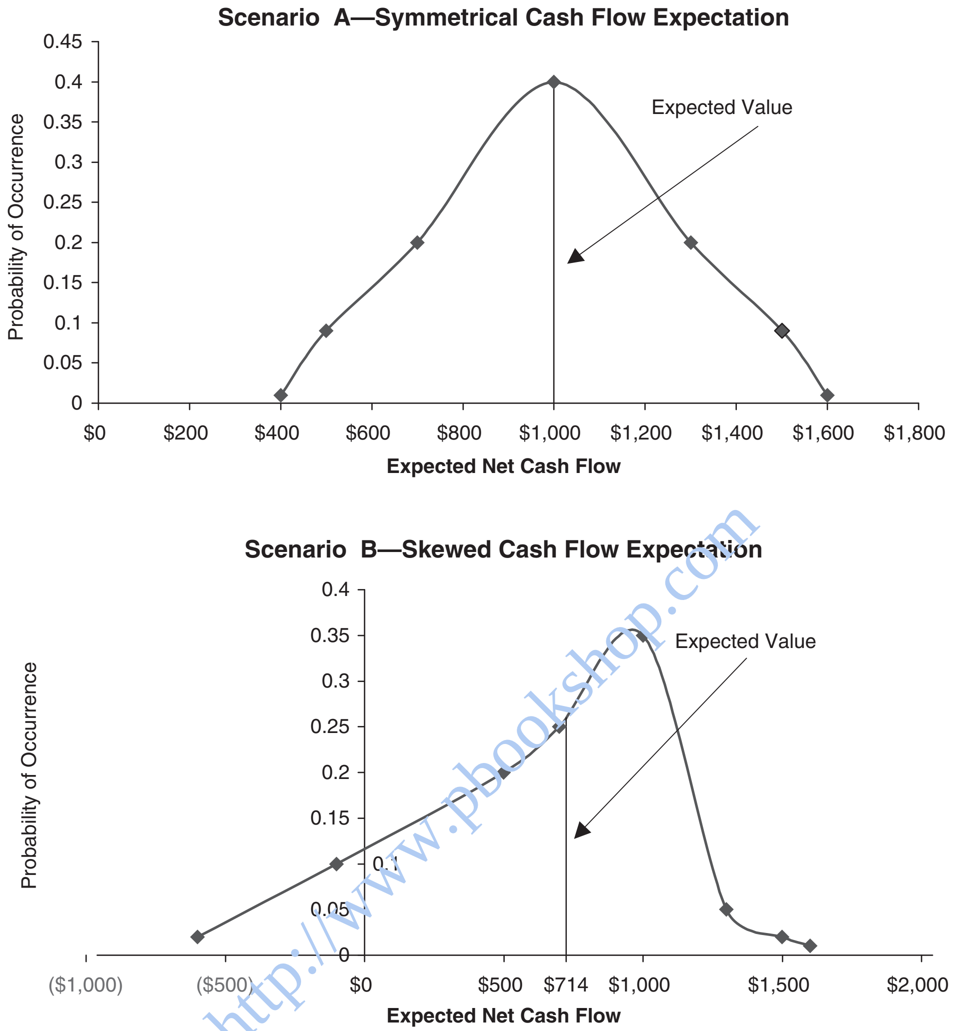

EXHIBIT 1.2 Example of Net Cash Flow Expectations (graphs of data from Exhibit 1.1)

As investment is made to increase capacity, the entire distribution of expected net cash flows can be ratcheted upward. But in any year in the future, the distribution of possible net cash flow may be and likely is skewed. So even if one does not receive or develop a probability distribution of net cash flows, the analyst should be aware that there is often more downside risk than upside potential in any year's net cash flows for many businesses.

As we pointed out in Formula 1.1, in calculating the present value of economic benefits, the numerator is the expected economic benefits. We have suggested that net cash flow is the preferred measure of economic income. While this is not a book on forecasting, the analyst may need to facilitate the preparation of expected net cash flows and/or test the reasonableness of the projected net cash flows provided.

Why Net Cash Flow Is the Preferred Measure of Economic Income

There are two reasons why the financial community tends to focus on net cash flow as the preferred measure of economic income to be discounted by the opportunity cost of capital. They are:

1. *Conceptual.* Net cash flow provides amounts that are available to compensate providers of capital for their investments in a discrete period of time. In a valuation context, it is important that the numerator of Formula 1.1 gives the most accurate estimate of what the business expects to generate as a return on the capital invested.
2. *Empirical.* It is the economic income measure that best matches discount rate estimates.

The case for preferring what they term *free cash flows* (i.e., net cash flows after tax) as the appropriate economic income measure to discount is clearly stated in Morningstar's *Stocks, Bonds, Bills and Inflation (SBBI) 2009 Valuation Edition Yearbook*:

> Several things can be noted about free cash flow. First, it is an after-tax concept. . . . Secondly, pure accounting adjustments need to be added back into the analysis. . . . Finally, cash flows necessary to keep the company going forward must be subtracted from the equation. These cash flows represent necessary capital expenditures to maintain plant, property, and equipment or other capital expenditures that arise out of the ordinary course of business. Another common subtraction is reflected in changes in working capital. The assumption in most business valuation settings is that the entity in question will remain a long-term going concern that will grow over time. As companies grow, they accumulate additional accounts receivable and other working capital elements that require additional cash to support.
>
> Free cash flow is the relevant cash flow stream because it represents the broadest level of earnings that can be generated by the asset. With free cash flow as the starting point, the owners of a firm can decide how much of the cash flow stream should be diverted toward new ventures, capital expenditures, interest payments, and dividend payments. It is incorrect to focus on earnings as the cash flow stream to be valued because earnings contain a number of accounting adjustments and already include the impact of the capital structure.[4]

If one uses the *SBBI* data or the data contained in the Duff & Phelps *Risk Premium Report* to develop a common equity discount rate—using either the build-up model or the Capital Asset Pricing Model (CAPM)—the discount rate is applicable to net cash flow available to the common equity investor because the *SBBI* and Duff & Phelps return data have two components:

[4] *SBBI Valuation Edition Yearbook* (Chicago: Morningstar, 2009), 13–14.

1. Dividends to the common stock
2. Changes in common stock prices

The investor receives the dividends, so their utilization is entirely at the investor's discretion. To the extent that net cash flows are retained in the business, they are assumed to be reinvested for the benefit of the common equity and added to the value of the common equity. For actively traded stock, the investor's realization of the change in stock price is equally discretionary because the stocks are highly liquid (i.e., they can be sold at their market price at any time, with the seller receiving the proceeds in cash within three business days).

DISCOUNT RATE IS NOT THE SAME AS CAPITALIZATION RATE

Because some practitioners and their clients confuse the terms, we point out here that discount rate and capitalization rate are two distinctly different concepts. *Discount rate* equates to *cost of capital*. The cost of capital is used as a discount rate to discount a stream of future economic income to a present value. This valuation process is called *discounting*.

In discounting, we project *all* expected economic income (cash flows or other measure of economic income) from the subject investment to the respective class or classes of capital over the life of the investment. Thus, the percentage return that we call the discount rate represents the total compound rate of return that an investor in that class of investment requires over the life of the investment.

There is a related process for estimating present value, which we call *capitalizing*. In capitalizing, instead of projecting all future economic income to the respective class(es) of capital, we focus on the economic income of one single period, usually the economic income expected in the first year immediately following the valuation date. That amount is assumed to represent the long-term sustainable base level of economic income. We then divide that single year economic income by a divisor called the *capitalization rate*.

As will be seen, the process of capitalizing is really just a shorthand form of discounting. The capitalization rate, as used in the income approach to valuation or project selection, is derived from the discount rate. (This differs from the market approach to valuation, where capitalization rates for various economic income measures are implied by taking the inverse of pricing multiples (e.g., inverting the price-to-earnings ratio).

A common error is the use of a discount rate as a capitalization rate. This is correct only if the expected cash flows are the same from the year following the valuation date into perpetuity (i.e., zero percent growth), as in the case of a perpetual preferred stock. The balance of this chapter presents the differences between discounting and capitalizing and alternative discounting and capitalizing conventions.

Capitalization Formula

Putting the capitalization concept into a formula, we have:

(Formula 1.4)

$$PV = \frac{NCF_1}{c}$$

where: PV = present value
NCF_1 = net cash flow expected in the first period immediately following the valuation date
c = capitalization rate

Functional Relationship between Discount Rate and Capitalization Rate

Investors often expect some level of growth over time in the cash flows available to pay dividends or distributions. Even if unit volume is expected to remain constant (i.e., no real growth), investors still might expect cash flows to grow at a rate approximating expected inflation. If the expected growth in cash flows for the investment is stable and sustainable over a long period of time, then the discount rate (cost of capital) can reasonably be converted to a capitalization rate.

The capitalization rate is a function of the discount rate. This raises the obvious question: What is the functional relationship between the discount rate and the capitalization rate?

Assuming stable long-term growth in cash flows from the subject investment, the capitalization rate equals the discount rate minus the expected long-term growth rate. This functional relationship can be stated as:

(Formula 1.5)

$$c = k - g$$

where: c = capitalization rate
k = discount rate (cost of capital) for the subject investment
g = expected long-term sustainable growth rate in the cash flow from the subject investment

The critical assumption in this formula is that the growth in the cash flow from the investment is relatively constant over the long-term (technically, into perpetuity).

Now we know two essential things about using the cost of capital to estimate present value using the capitalization method for a business, assuming relatively stable long-term growth in the net cash flow:

1. Present value equals the next period's expected net cash flow divided by the capitalization rate.
2. The net cash flow capitalization rate is the discount rate (cost of capital) minus the expected long-term rate of growth in the net cash flow. (Technically, growth in this context means into perpetuity. However, after 15 or 20 years, the remaining rate of growth has minimal impact on the present value, due to very small present value factors of more distant future years.) The growth in net cash flow is sustainable because one has subtracted the investments needed to realize the expected revenues and expenses.

We can combine these two relationships into a single formula as:

(Formula 1.6)

$$PV = \frac{NCF_1}{k - g}$$

where: PV = present value
NCF_1 = net cash flow expected in period 1, the period immediately following the valuation date
k = discount rate (cost of capital)
g = expected long-term growth rate in net cash flow

A simple example of substituting numbers into Formula 1.6 is an equity investment with a constant expected growth in net cash flow. Let us make three assumptions:

1. The net cash flow in period 1 is expected to be $100.
2. The cost of capital (i.e., the market-required total return or the discount rate) for this investment is estimated to be 13%.
3. The sustainable rate of long-term growth in net cash flow from year 1 to perpetuity is expected to be 3%.

Substituting numbers from the preceding assumptions into Formula 1.6 gives us:

(Formula 1.7)

$$PV = \frac{\$100}{0.13 - 0.03} = \frac{\$100}{0.10} = \$1{,}000$$

In this example, the estimated value of the investment in the business is $1,000.

Major Difference between Discounting and Capitalizing

From the preceding discussion, we can now deduce a critical insight: The difference between discounting and capitalizing is in how we reflect changes over time in expected future cash flows.

In *discounting*: Each future change in cash flow is estimated specifically and included in the numerator.

In *capitalizing*: Estimates of rates of changes in future cash flows are averaged into one annually compounded growth rate, which is then subtracted from the discount rate in the denominator.

If we assume that there really is a constant compounded growth rate in cash flow from the investment into perpetuity, then it is a mathematical truism that the discounting method and the capitalizing method will produce identical values.

Constant Growth or Gordon Growth Model

One frequently encountered minor modification to Formula 1.6 is to use as the base period the period just completed prior to the valuation date, instead of next period's estimate. The assumption is that net cash flows will grow evenly into perpetuity from the period immediately preceding the valuation date. This constant growth capitalization formula, commonly known as the Gordon Growth Model (named for Professor Myron Gordon, who popularized this formulation), as applied to the net cash flow is as follows:

(Formula 1.8)

$$PV = \frac{NCF_0(1+g)}{k-g}$$

where: PV = present value
NCF_0 = net cash flow in period 0, the period immediately preceding the valuation date
k = discount rate (cost of capital)
g = expected sustainable long-term growth rate in net cash flow

Note that for this model to make economic sense, NCF_0 must represent a normalized amount of net cash flow from the investment for the previous year, from which a steady rate of growth is expected to proceed. Therefore, NCF_0 need not be the actual net cash flow for period 0 but may reflect certain normalization adjustments, such as elimination of the effect of one or more nonrecurring factors. In fact, if NCF_0 *is* the actual net cash flow for period 0, the valuation analyst must take reasonable steps to be satisfied that NCF_0 is indeed the most reasonable base from which to start the expected growth embedded in the growth rate. Any valuation report prepared should state the steps taken and the assumptions made in concluding that last year's actual results are the most reasonable base for expected net cash flow growth. Mechanistic acceptance of recent results as representative of future expectations is one of the most common errors in implementing the capitalization method of valuation.

For a simple example of the use of Formula 1.8, accept all assumptions in the previous example, with the exception that the $100 net cash flow expected in period 1 is instead the normalized base cash flow for period 0. (The $100 is for the period just ended, rather than the expectation for the period just starting.) Substituting the numbers with these assumptions into Formula 1.8 produces:

(Formula 1.9)

$$\begin{aligned} PV &= \frac{\$100(1+0.03)}{0.13-0.03} \\ &= \frac{\$103}{0.10} \\ &= \$1{,}030 \end{aligned}$$

In this example, the estimated value of the investment is $1,030. The relationship between this and the previous example is simple and straightforward. We moved the receipt of the $100 back in time by one period, and the value of the investment was increased by 3%, the growth rate. In a constant growth capitalization model, even assuming that all the net cash flows are distributed, the value of the investment grows at the same rate as the rate of growth of the cash flows. The reason is that, in defining net cash flow (as we did in the previous chapter); we have already subtracted the amount of capital expenditures and additions to net working capital necessary to sustain the projected growth.

The investor in this example thus earns a *total* rate of return of 13%, comprised of 10% current return (the capitalization rate) plus 3% annually compounded growth in the value of the investment.

Combining Discounting and Capitalizing (Two-Stage Model)

For many investments, even given an accurate estimate of the cost of capital, there are practical problems with either pure discounting or pure capitalizing methods of valuing expected net cash flows.

- *Problem with discounting*. There are few equity investments for which returns for each specific incremental period can be projected with accuracy many years into the future.
- *Problem with capitalizing*. For most equity investments, it is not reasonable to expect a constant growth rate into perpetuity from either the year preceding or the year following the valuation date.

This dilemma typically is dealt with by combining the discounting method and the capitalizing method into a *two-stage model*. The idea is to project discrete cash flows for some number of periods into the future and then to project a steady growth model starting at the end of the discrete projection period. Each period's expected discrete cash flow is discounted to a present value, and the capitalized value of the projected cash flows following the end of the discrete projection period is also discounted back to a present value. The sum of the present values is the total present value. The capitalized value of the projected cash flows following the discrete projection period is called the *terminal value* or *residual value*.

The preceding narrative explanation of a two-stage model is summarized in seven steps:

Step 1. Determine a reasonable length of time for which discrete projections of net cash flows can be made.

Step 2. Estimate specific expected net cash flows for each of the discrete projection periods.

Step 3. Estimate a sustainable long-term rate of growth in net cash flows from the end of the discrete projection period forward.

Step 4. Use the constant growth model (Gordon Growth Model; Formula 1.8) to estimate the future value as of the end of the discrete projection period (commonly referred to as the terminal or residual value).

Step 5. Discount each of the discrete net cash flows back to their present value at the discount rate (cost of capital) for the number of periods until it is projected to be received.

Step 6. Discount the terminal value (estimated in step 4) back to a present value for the number of periods in the discrete projection period (the same number of periods as the last discrete net cash flow).

Step 7. Sum the value derived from steps 5 and 6.

These steps can be summarized by the next formula, which assumes that net cash flows are received at the end of each year:

(Formula 1.10)

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \cdots + \frac{NCF_n}{(1+k)^n} + \frac{\frac{NCF_n(1+g)}{k-g}}{(1+k)^n}$$

where: PV = present value

$NCF_1 \ldots NCF_n$ = net cash flow expected in each of the periods 1 through n, n being the last period of the discrete net cash flow projections

k = discount rate (cost of capital)

g = expected sustainable long-term growth rate in net cash flow, starting with the last period of the discrete projections as the base year

The discrete projection period in the two-stage model depends on how many years or periods there will be variable change in net cash flows. The residual period begins whenever the net cash flow begins growing at a constant growth rate. Having said this, it is not uncommon for the discrete periods to be as few as 3 years or as many as 10 years, while for cyclical businesses, the discrete period commonly corresponds to the number of years or periods until the point is reached where the net cash flow represents an average base net cash expected over an entire business cycle. For simplicity in applying Formula 1.10, we will just use a three-year discrete projection period. Let us make three assumptions:

1. Expected net cash flows for years 1, 2, and 3 are \$100, \$120, and \$140, respectively.
2. Beyond year 3, based on the business's performance and industry and overall economic expectations, 5% average growth in net cash flow appears to be a reasonable estimate of sustainable long-term growth.
3. The cost of capital for this investment is estimated to be 12%.

Substituting numbers derived from these assumptions into Formula 1.10 produces:

(Formula 1.11)

$$
\begin{aligned}
PV &= \frac{\$100}{(1+0.12)} + \frac{\$120}{(1+0.12)^2} + \frac{\$140}{(1+0.12)^3} + \frac{\frac{\$140(1+0.05)}{0.12-0.05}}{(1+0.12)^3} \\
&= \frac{\$100}{1.12} + \frac{\$120}{1.2544} + \frac{\$140}{1.4049} + \frac{\frac{\$147}{0.07}}{1.4049} \\
&= \$89.30 + \$95.66 + \$99.65 + \frac{\$2,100}{1.4049} \\
&= \$89.30 + \$95.66 + \$99.65 + \$1,494.77 \\
&= \$1,779.38
\end{aligned}
$$

Thus, the estimated value of this investment is $1,779.

The above formula is frequently presented in tabular form, as follows:

Period	Cash Flow		Factor Multiplier		Present Value
1	$100	×	.8929	=	$89.29
2	120	×	.7972	=	95.66
3	140	×	.7118	=	99.65
Terminal Value	2,100	×	.7118	=	1,494.78
					$1,779.38

Equivalency of Discounting and Capitalizing Models

If certain assumptions are met, the discounting and capitalizing methods of using the cost of capital will produce identical estimates of present value. Let us test this on the example used in Formula 1.6. Recall that we assumed net cash flow in period 1 of $100, growing into perpetuity at 3%. The cost of capital (discount rate) was 13%, so we subtracted the growth rate of 3% to get a capitalization rate of 10%. Capitalizing the $100 (period 1 expected net cash flow) at 10% gave us an estimated present value of $1,000 ($100 ÷ 0.10 = $1,000).

Let us take these same assumptions and put them into a discounting model. For simplicity, we will only use three periods for the discrete projection period, but it would not make any difference how many discrete projection periods we used.

(Formula 1.12)

$$
\begin{aligned}
PV &= \frac{\$100}{(1+0.13)} + \frac{\$100(1.03)}{(1+0.13)^2} + \frac{\$100(1.03)^2}{(1+0.13)^3} + \frac{\frac{\$100(1.03)^3}{0.13-0.03}}{(1.13)^3} \\
&= \frac{\$100}{1.13} + \frac{\$103}{1.2769} + \frac{\$106.09}{1.4429} + \frac{\frac{\$109.27}{0.10}}{1.4429} \\
&= \$88.50 + \$80.66 + \$75.53 + \frac{\$1092.73}{1.4429} \\
&= \$88.50 + \$80.66 + \$73.53 + \$757.31 \\
&= \$1,000
\end{aligned}
$$

This example, showing the equivalency of using the cost of capital in either the discounting or the capitalizing model, when certain key assumptions are met, demonstrates the point that capitalizing is really just a shorthand form of discounting. Capitalization is often used when one believes that the current sustainable net cash flow will grow at an average growth rate in the future or if one does not have sufficient information to implement a discounting model but nevertheless feels comfortable that capitalizing a single year's net cash flow will provide meaningful valuation results. Nevertheless, when using a capitalization of net cash flow model, the analyst should consider whether the present value of net cash flows would be the same if a full discounting model were used. If not, it may be propitious to review and possibly adjust certain assumptions. If the discounting and capitalization of net cash flow models produce different answers using the same cost of capital and the same inputs, there may be some kind of an internal inconsistency.

Caveat: We have seen instances where analysts have used both a discounted cash flow method and the single-year capitalization of net cash flow method in the same valuation analysis and then weighed the two results in the reconciliation of value. This is not correct. In developing an income approach the analyst should determine whether a multi-period discounted cash flow is needed, or whether the abridged single-year capitalization method will suffice.

COST OF CAPITAL SHOULD REFLECT THE RISK OF THE INVESTMENT

The cost of capital for any given investment is a combination of two basic factors:

A *risk-free rate*. By "risk-free rate," we mean a rate of return that is available in the market on an investment that is free of default risk, usually the yield to maturity on a U.S. government security. It is a "nominal" rate (i.e., it includes expected inflation).

A *premium for risk*. This is an expected amount of return over and above the risk-free rate to compensate the investor for accepting risk (e.g., risk of amount and timing of net cash flows, and liquidity of the asset).

The generalized cost of capital relationship is:

(Formula 1.13)

$$E(R_i) = R_f + RP_i$$

where: $E(R_i)$ = expected return of asset i
R_f = risk-free rate
RP = risk premium for asset i

Quantifying the amount by which risk affects the cost of capital for any particular business or investment is arguably one of the most difficult analyses in the field of corporate finance, including valuation and capital budgeting. Estimating the cost of capital is first and foremost an exercise in pricing risk.

Defining Risk

Probably the most widely accepted definition of risk in the context of business valuation is *the degree of uncertainty (or lack thereof) of achieving future expectations at the times and in the amounts expected*.[5] The definition implies uncertainty as to both the amounts and the timing of *expected economic income*. By expected economic income, in a technical sense, we mean the expected value (i.e., mean or average) of the probability distribution of possible economic income for each forecast period. The point to understand here is that the uncertainty encompasses the full distribution of possible economic income for each period both above and below the expected value.

Inasmuch as uncertainty is within the mind of the individual investor, we cannot measure the risk directly. Consequently, participants in the financial markets have developed ways of measuring factors that investors normally would consider in their effort to incorporate risk into their required rate of return.

Throughout this book we equate risk with uncertainty, as does most related literature. However, some analysts make a useful distinction between the two terms. That is, risk is present where the parameters of uncertainty are defined (i.e., when the generating function is known with certainty), as in a coin toss (e.g., if forecasters all agree that recession will occur next year then the subject business's net cash flows will still vary, but within the forecast of recession). Uncertainty beyond risk occurs when analysts have the possibility of an infinite number of subjective inputs (e.g., wide divergence of opinions among forecasters as to whether there will be a recession next year or not).[6]

No matter how many probability distributions or Monte Carlo simulations are used to create a financial forecast, all risk cannot be eliminated. Therefore, projected net cash flows cannot be discounted at the risk-free rate.

How Risk Impacts the Cost of Capital

The cost of capital for any given investment is a combination of two basic factors: a *risk-free rate,* R_f, and a *premium for risk*, *RP*.

As the market's perception of the degree of risk of an investment increases, the risk premium, *RP*, increases so that the rate of return that the market requires (the discount rate) increases for a given set of expected cash flows. The greater the market's required rate of return, the less is the present value of the investment; the less the market's required rate of return, the greater is the present value of the investment.

Risk is a major concern of investors. The risk-free rate compensates investors for renting out their money (i.e., for delaying consumption over some future time period with a return of currency with less purchasing power in the future). This component of the cost of capital is readily observable in the marketplace and

[5] David Laro and Shannon P. Pratt, *Business Valuation and Taxes: Procedure, Law, and Perspective*, 2nd ed. (Hoboken, NJ: John Wiley & Sons, 2010), Chapter 12.

[6] Evan W. Anderson, Eric Ghysels, and Jennifer L. Juergens, "The Impact of Risk and Uncertainty on Expected Returns," Working paper, June 22, 2009. Available at http://ssrn.com/abstract=890621.

generally differs from one investment to another only to the extent of the time horizon (maturity) selected for measurement of the risk-free rate.

The risk premium results from the uncertainty of expected returns and varies widely from one prospective capital investment to another. We could say that the market abhors uncertainty and consequently requires a high rate of return to accept uncertainty. Since uncertainty as to timing and amounts of future net cash flow is greatest for equity investors, the high risk requires equity investors as a class of providers of capital to have the greatest cost of capital.

Risk Aversion versus Risk Neutrality Earlier we discussed that one should discount or capitalize the statistical expected value of net cash flows. One can think of any one year's distribution of possible net cash flows as a bundle of possible outcomes (sometimes termed *contingent claims* on the asset). The present value of this series of contingent claims can be depicted in the following formula:

(Formula 1.14)

$$PV = \sum_{1}^{n} \frac{E(cash\ flow)_n}{(1+k)^n}$$

If investors were risk neutral, the appropriate discount rate for estimating the present value of the expected cash flows would be the risk-free rate.

What is risk neutral? Assume the investor is risk neutral and that the return on the investment is expected to be received one year from the date of the investment. Assume that the possible payoff in one year equals the expected value of the cash flows. Investors would be satisfied with a payoff equal to the present value of expected cash flows calculated at the risk-free rate because the expected cash flows represent a *fair bet*. The investor pays an amount equal to the present value of the expected net cash flows discounted at the risk-free rate (which takes into account the time-value of money for the one year period of the investment) and the investor receives the opportunity to realize one of the possibilities of net cash outcomes. The expected payoff is exactly equal to the possible net cash flow outcome multiplied by the probability that the net cash flow outcome will occur.

But investors are not risk neutral; in the literature investors are generally assumed to be risk averse. Risk aversion is equivalent to paying more attention to unpleasant outcomes, relative to their actual probability of occurrence. Generally, investors are more concerned about losing an amount of money than the possibility of making the same amount of money. Exhibit 1.3 helps explain the concept of risk aversion.

Scenario A represents the expected net cash flow from a risk-free investment. Assume that the investor could buy the investment today and be guaranteed \$1,000 in one year. The expected net cash flow equals \$1,000 in one year. An investor would be willing to pay an amount for this investment opportunity and require a return that compensates him for the time-value of money; that is, the rate of return that compensates the investor for his preference of holding money today versus one year hence (but with no risk of loss). Assuming a risk-free rate of, say, 5%, the present value of the expected net cash flow equals \$952 and the market price could be expected to be approximately \$952.

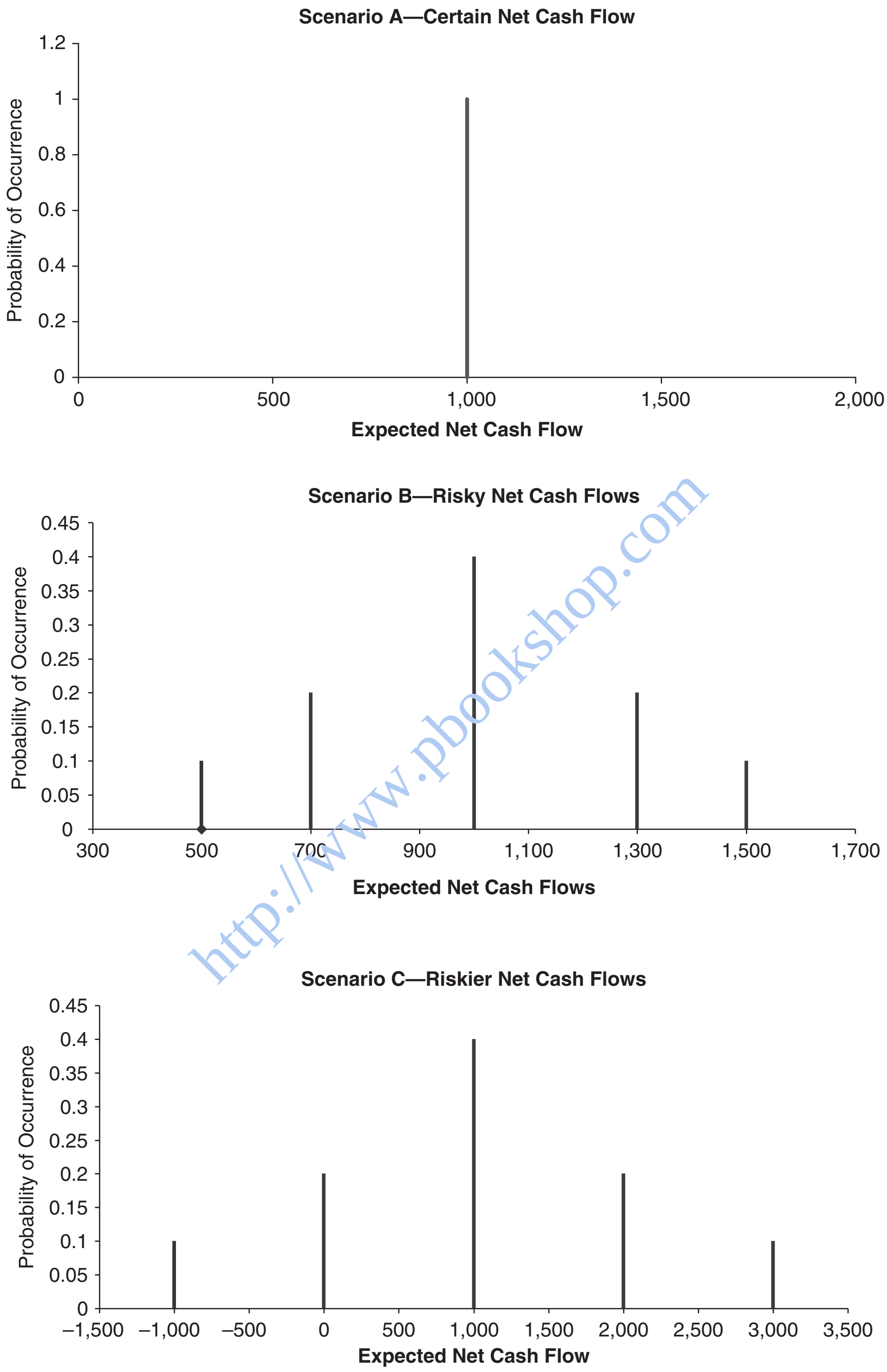

EXHIBIT 1.3 Valuation of Expected Net Cash Flows with Varying Distributions

Scenario B represents the expected cash flows from a risky investment. The expected net cash flow in one year is again equal to $1,000 but there is a chance that the net cash flows will be less than $1,000 or greater than $1,000. If the investor were risk neutral, he would be willing to pay $952 for the fair bet to earn more than $1,000 (up to $1,500) or less than $1,000 (as little as $500). But investors are not risk neutral. They want to be compensated for the chance that they could end up with only $500. The investor would require a greater rate of return than in Scenario A because investors are risk averse and want to be compensated for the risk by a greater rate of return. Let's assume that the market prices the investment opportunity in Scenario B at $870. This yields an expected rate of return equal to approximately 15%.

Scenario C represents the expected net cash flows from an even riskier investment. The expected net cash flow in one year is again equal to $1,000, but there is even a greater chance that the net cash flows will be less than $1,000 or greater than $1,000. If the investor were risk neutral, he would be willing to pay $952 for the fair bet to earn more than $1,000 (up to $3,000) or less than $1,000 (a loss of $1,000). But as investors are not risk neutral, they want to be compensated for the chance that they could end up losing $1,000. The investor would require a greater rate of return than in Scenario B because investors are risk averse and want to be compensated for the increased risk by an increased rate of return. Let's assume that the market prices the investment opportunity in Scenario C at $770, yielding an expected rate of return equal to approximately 30%.

The appropriate discount rate for discounting risky net cash flows is not a risk-free rate of return. Would the market only demand the risk-free rate of return for taking on the variability of the net cash flows? The answer is no. The market will demand compensation (added return) for accepting the risk that the actual net cash flows will differ from the expected net cash flows in future periods and the added return will increase depending on the amount of expected dispersion that could occur. That is, one would expect that the greater the dispersion of expected net cash flows the greater the discount rate.[7]

Market Returns Increase as Risk Increases by Asset Class Because investors are risk averse, the market requires an increasing rate of return as the risk of a bad outcome increases, even if the expected net cash flow is identical in all three scenarios. How do we know the market demands and receives greater returns for taking on greater risk? If one looks across asset classes at mean returns and risk (as measured by the standard deviation of returns realized over time), one observes that greater returns seem closely related to greater risk (see Exhibit 1.4).[8]

In fact, if one plots the observed relationship of risk and returns over time (as compiled in Exhibit 1.4), one observes a strong linear relationship between risk and return, which is referred to as the Capital Market Line. The Capital Market Line is defined as a line used in the Capital Asset Pricing Model that plots the rates of return for efficient portfolios, depending on the rate of return and the level of risk (standard

[7] If one converts the actual probabilities to what is termed risk-neutral probabilities, the risk-free rate is the correct discount rate.

[8] William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* (September 1964): 425–442.

EXHIBIT 1.4 Returns and Standard Deviation of Returns by Asset Class for 1926–2008

1926–2008	Arithmetic Mean Returns	Standard Deviation of Returns
Large Company Stocks	11.7%	20.6%
Ibbotson Small Company Stocks	16.4%	33.0%
Mid-Cap Stocks	13.4%	24.9%
Low-Cap Stocks	14.9%	29.4%
Micro-Cap Stocks	17.7%	39.2%
Ibbotson Long-Term Corporate Bonds	6.2%	8.4%
Ibbotson Long-Term Government Bonds	6.1%	9.4%
Treasury Bills	3.8%	3.1%

Source: Compiled from data in *Stocks, Bonds, Bills and Inflation 2009 Yearbook*. Copyright 2009 Morningstar, Inc. All rights reserved. Used with permission.

deviation) for a particular portfolio.[9] The empirical estimate of the Capital Market Line shows the market's pricing of portfolios of assets over the period 1926 through 2008. As the risk (standard deviation of returns) increases, the realized return increases as shown in Exhibit 1.5.

The observed relationship between risk and return indicates that the market is relatively efficient over the long term. Assets with greater risk are priced to realize greater returns.

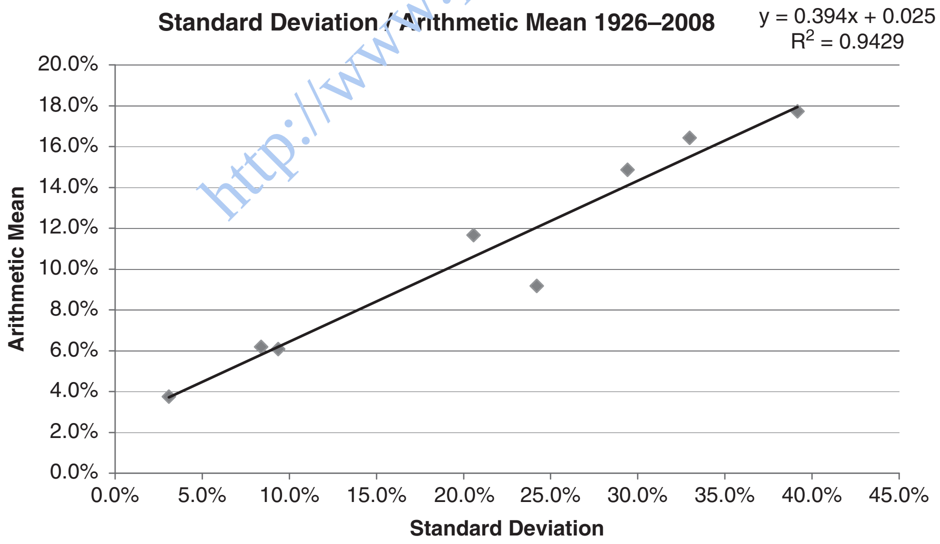

EXHIBIT 1.5 Capital Market Line—Empirical Estimate Based on Realized Returns by Asset Class

[9] The Capital Market Line is different than the Security Market Line (SML).