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CHAPTER 1
Introduction

Finance is the application of economic principles to decision making, and involves the allocation of money under conditions of uncertainty. Investors allocate their funds among financial assets in order to accomplish their objectives. Business entities and government at all levels raise funds by issuing claims in the form of debt (e.g., loans and bonds) or equity (e.g., common stock) and, in turn, invest those funds. Finance provides the framework for making decisions as to how those funds should be obtained and then invested.

The field of finance has three specialty areas: (1) capital markets and capital market theory, (2) financial management, and (3) portfolio management. The specialty field of *capital markets* and *capital market theory* focuses on the study of the financial system, the structure of interest rates, and the pricing of risky assets. *Financial management*, sometimes called *business finance*, is the specialty area of finance concerned with financial decision making within a business entity. Although we often refer to financial management as *corporate finance*, the principles of financial management also apply to other forms of business and to government entities. Moreover, not all nongovernment business enterprises are corporations. Financial managers are primarily concerned with investment decisions and financing decisions within business. Making investment decisions that involve long-term capital expenditures is called *capital budgeting*. *Portfolio management* deals with the management of individual or institutional funds. This specialty area of finance—also commonly referred to as *investment management, asset management*, and *money management*—involves selecting an investment strategy and then selecting the specific assets to be included in a portfolio.

A critical element common to all three specialty areas in finance is the concept of risk. Measuring and quantifying risk is critical for the fair valuation of an asset, the selection of capital budgeting projects in financial management, the selection of individual asset holdings, and portfolio construction in portfolio management. The field of *risk management* includes

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the identification, measurement, and control of risk in a business entity or a portfolio.

Sophisticated mathematical tools have been employed in order to deal with the risks associated with individual assets, capital budgeting projects, and selecting assets in portfolio construction. The use of such tools is now commonplace in the financial industry. For example, in portfolio management, practitioners run statistical routines to identify risk factors that drive asset returns, scenario analyses to evaluate the risk of their positions, and algorithms to find the optimal way to allocate assets or execute a trade.

This book focuses on two quantitative tools—optimization and simulation—and discusses their applications in finance. In this chapter, we briefly introduce these two techniques, and provide an overview of the structure of the book.

OPTIMIZATION

Optimization is an area in applied mathematics that, most generally, deals with efficient algorithms for finding an optimal solution among a set of solutions that satisfy given constraints. The first application of optimization in finance was suggested by Harry Markowitz in 1952, in a seminal paper that outlined his mean-variance optimization framework for optimal asset allocation. Some other classical problems in finance that can be solved by optimization algorithms include:

- Is there a possibility to make riskless profit given market prices of related securities? (This opportunity is called an *arbitrage opportunity* and is discussed in Chapter 13.)
- How should trades be executed so as to reach a target allocation with minimum transaction costs?
- Given a limited capital budget, which capital budgeting projects should be selected?
- Given estimates for the costs and benefits of a multistage capital budgeting project, at what stage should the project be expanded/abandoned?

Traditional optimization modeling assumes that the inputs to the algorithms are certain, but there is also a branch of optimization that studies the optimal decision under uncertainty about the parameters of the problem. Fast and reliable algorithms exist for many classes of optimization problems, and advances in computing power have made optimization techniques a viable and useful part of the standard toolset of the financial modeler.

SIMULATION

Simulation is a technique for replicating uncertain processes, and evaluating decisions under uncertain conditions. Perhaps the earliest application of simulation in finance was in financial management. Hertz (1964) argued that traditional valuation methods for investments omitted from consideration an important component: the fact that many of the inputs were inaccurate. He suggested modeling the uncertainty through probability-weighted scenarios, which would allow for obtaining a range of outcomes for the value of the investments and associated probabilities for each outcome. These ideas were forgotten for a while, but have experienced tremendous growth in the last two decades. Simulation is now used not only in financial management, but also in risk management and pricing of different financial instruments. In portfolio management, for example, the correlated behavior of different factors over time is simulated in order to estimate measures of portfolio risk. In pricing financial options or complex securities, such as mortgage-backed securities, paths for the underlying risk factors are simulated; and the fair price of the securities is estimated as the average of the discounted payoffs over those paths. We will see numerous examples of such simulation applications in this book.

Simulation bears some resemblance to an intuitive tool for modifying original assumptions in financial models—what-if analysis—which has been used for a long time in financial applications. In *what-if analysis*, each uncertain input in a model is assigned a range of possible values—typically, best, worst, and most likely value—and the modeler analyzes what happens to the decision under these scenarios. The important additional component in simulation modeling, however, is that there are probabilities associated with the different outcomes. This allows for obtaining an additional piece of information compared to what-if analysis: the probabilities that specific final outcomes will happen. Probability theory is so fundamental to understanding the nature of simulation analysis, that we include a chapter (Chapter 3) on the most important aspects of probability theory that are relevant for simulation modeling.

OUTLINE OF TOPICS

The book is organized as follows. Part One (Chapters 2 through 6) provides a background on the fundamental concepts used in the rest of the book. Part Two (Chapters 7 through 10) introduces the classical underpinnings of modern portfolio theory, and discusses the role of simulation and optimization in recent developments. Part Three (Chapters 11 and 12)

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summarizes important models for asset pricing and asset price dynamics. Understanding how to implement these models is a prerequisite for the material in Part Four (Chapters 13 through 16), which deals with the pricing of financial derivatives, mortgage-backed securities, advanced portfolio management, and advanced simulation methods. Part Five (Chapters 17 and 18) discusses applications of simulation and optimization in capital budgeting and real option valuation. The four appendices (on the companion web site) feature introductions to linear algebra concepts, @RISK, MATLAB, and Visual Basic for Applications in Microsoft Excel.

We begin by listing important finance terminology in Chapter 2. This includes basic theory of interest; terminology associated with equities, fixed income securities, and trading; calculation of rate of return; and useful concepts in fixed income, such as spot rates, forward rates, yield, duration, and convexity.

Chapter 3 is an introduction to probability theory, distributions, and basic statistics. We review important probability distributions, such as the normal distribution and the binomial distribution, measures of central tendency and variability, and measures of strength of codependence between random variables. Understanding these concepts is paramount to understanding the simulation models discussed in the book.

Chapter 4 introduces simulation as a methodology. We discuss determining inputs for and interpreting output from simulation models, and explain the methodology behind generating random numbers from different probability distributions. We also touch upon recent developments in efficient random number generation, which provides the foundation for the advanced simulation methods for financial derivative pricing discussed in Part Four of the book.

In Chapter 5 we provide a practical introduction to optimization. We discuss the most commonly encountered types of optimization problems in finance, and elaborate on the concept of "difficult" versus "easy" optimization problems. We introduce optimization duality and describe intuitively how optimization algorithms work. Illustrations of simple finance problems that can be handled with optimization techniques are provided, including examples of optimal portfolio allocation and cash flow matching from the field of portfolio management, and capital budgeting from the field of financial management. We also discuss dynamic programming—a technique for solving optimization problems over multiple stages. Multistage optimization is used in Chapters 13 and 18. Finally, we review available software for different types of optimization problems and portfolio optimization in particular.

Classical optimization methods treat the parameters in optimization problems as deterministic and accurate. In reality, however, these parameters are typically estimated through error-prone statistical procedures or

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based on subjective evaluation, resulting in estimates with significant estimation errors. The output of optimization routines based on poorly estimated inputs can be at best useless and at worst seriously misleading. It is important to know how to treat uncertainty in the estimates of input parameters in optimization problems. Chapter 6 provides a taxonomy of methods for optimization under uncertainty. We review the main ideas behind dynamic programming under uncertainty, stochastic programming, and robust optimization, and illustrate the methods with examples. We will encounter these methods in applications in Chapters 9, 13, 14, and 18.

Chapter 7 uses the concept of optimization to introduce the mean-variance framework that is the foundation of modern portfolio theory. We also present an alternative framework for optimal decision making in investments—expected utility maximization—and explain its relationship to mean-variance optimization.

Chapter 8 extends the classical mean-variance portfolio optimization theory to a more general mean-risk setting. We cover the most commonly used alternative risk measures that are generally better suited than variance for describing investor preferences when asset return distributions are skewed or fat-tailed. We focus on two popular portfolio risk measures—value-at-risk and conditional value-at-risk—and show how to estimate them using simulation. We also formulate the problems of optimal asset allocation under these risk measures using optimization.

Chapter 9 provides an overview of practical considerations in implementing portfolio optimization. We review constraints that are most commonly faced by portfolio managers, and show how to formulate them as part of optimization problems. We also show how the classical framework for portfolio allocation can be extended to include transaction costs, and discuss index tracking, optimization of trades across multiple client accounts, and robust portfolio optimization techniques to minimize estimation error.

While Chapter 9 focuses mostly on equity portfolio management, Chapter 10 discusses the specificities of fixed income (bond) portfolio management. Many of the same concepts are used in equity and fixed income portfolio management (which are defined in Chapter 2); however, fixed income securities have some fundamental differences from equities, so the concepts cannot always be applied in the same way in which they would be applied for stock portfolios. We review classical measures of bond portfolio risk, such as duration, key rate duration, and spread duration. We discuss bond portfolio optimization relative to a benchmark index. We also give examples of how optimization can be used in liability-driven bond portfolio strategies such as immunization and cash flow matching.

Chapter 11 transitions from the topic of portfolio management to the topic of asset pricing, and introduces standard financial models for explaining asset returns—the Capital Asset Pricing Model (CAPM), which is based

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on the mean-variance framework described in Chapter 7, the Arbitrage Pricing Theory (APT), and factor models. Such models are widely used in portfolio management—they not only help to model the processes that drive asset prices, but also substantially reduce the computational burden for statistical estimation and asset allocation optimization algorithms.

Chapter 12 focuses on dynamic asset pricing models, which are based on random processes. We examine the most commonly used types of random walks, and illustrate their behavior through simulation. The models discussed include arithmetic, geometric, different types of mean-reverting random walks, and more advanced hybrid models. In our presentation in the chapter, we assume that changes in asset prices happen at discrete time intervals. At the end of the chapter, we extend the concept of a random walk to a random process in continuous time.

The concepts introduced in Chapter 12 are reused multiple times when we discuss valuation of complex securities and multistage investments in Parts Four and Five of the book. The first chapter in Part Four, Chapter 13, is an introduction to the topic of financial derivatives. It lists the main classes of financial derivative contracts (futures and forwards, options, and swaps), explains the important concepts of arbitrage and hedging, and reviews classical methods for pricing derivatives, such as the Black-Scholes formula and binomial trees.

Chapter 14 builds on the material in Chapter 13, but focuses mainly on the use of simulation for pricing complex securities. Some of the closed-form formulas provided in Chapter 12 and the assumptions behind them become more intuitive when illustrated through simulation of the random processes followed by the underlying securities. A large part of the chapter is dedicated to variance reduction techniques, such as antithetic variables, stratified sampling, importance sampling, and control variates, as well as quasi–Monte Carlo methods. Such techniques are widely used today for efficient implementation of simulations for pricing securities and estimating sensitivity to different market factors. We provide specific examples of these techniques, and detailed VBA and MATLAB code to illustrate their implementation.

The numerical pricing methods in Chapter 15 are based on similar techniques to the ones discussed in Chapter 14, but the context is different. We introduce a complex type of fixed-income securities—mortgage-backed securities—and discuss in detail a part of the simulation that is specific to fixed-income securities—generating scenarios for future interest rates and the entire yield curve.

Chapter 16 builds on Chapters 7, 8, 9, 13, and 14, and contains a discussion of how derivatives can be used for portfolio risk management and return enhancement strategies. Simulation is essential for estimating the risk of a portfolio that contains complex financial instruments, but the

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process can be very slow in the case of large portfolios. We highlight some numerical issues, standard simulation algorithms, and review methods that have been suggested for reducing the computational burden.

Chapters 17 and 18 cover a different area of finance—financial management—but they provide useful illustrations for the difference applying simulation and optimization makes in classical finance decision-making frameworks. Chapter 17 begins with a review of so-called discounted cash flow (DCF) methodologies for evaluating company investment projects. It then discusses (through a case study) how simulation can be used to estimate stand-alone risk and enhance the analysis of such projects.

Chapter 18 introduces the real options framework, which advocates for accounting for existing options in project valuation. (The DCF analysis ignores the potential flexibility in projects—it assumes that there will be no changes once a decision is made.) While determining the inputs for valuation of real options presents significant challenges, the actual techniques for pricing these real options are based on the techniques for pricing financial options introduced in Chapters 13 and 14. Simulation and multistage optimization can again be used as valuable tools in this new context.