

Index

- additive processes
 - see also* stochastic processes
 - concepts 60–77, 80–93
 - definition 60–1
- affine models, concepts 225–8
- algebraic dual space, definition 206
- analytic functions
 - see also* complex . . .
 - definition 179–80
- angular frequencies 117–18
- appendices 153–228
- APT *see* arbitrage pricing theory
- arbitrage opportunities, concepts 1–5, 79–93
- arbitrage pricing theory (APT), definition 80–1
- arbitrage-free pricing
 - concepts 79–93, 129–52
 - Lévy markets 92–3
- Argand diagrams *see* complex planes
- arrival-of-information probability laws, Lévy markets 6, 39–49, 57
- Arrow–Debreu securities 1, 7–12, 26–7, 85–93
 - see also* options . . .
- Asian options 146–52
 - see also* exotic . . . ; options . . .
- Asmussen–Rosinski theorem, definition 76–7
- asset prices
 - arrival-of-information probability laws 6, 39–49, 57
 - dynamics 3–6, 29–55
- asset-or-nothing call options
 - see also* digital . . .
 - concepts 2–3, 10–11, 86–93
- asset-or-nothing put options
 - see also* digital . . .
 - concepts 2–3, 10–11, 86–93
- associativity 201–2
- at-the-money options (ATM) 4–5, 11–12, 152
- attainable assets
 - see also* complete markets
 - concepts 3–4, 83–93
- autocorrelation, Fourier transform 119–28
- Bachelier, Louis 29, 30
- Banach spaces, concepts 166, 205
- Barndorff–Nielsen–Shephard model, definition 72–3
- Bernoulli distributions, concepts 36–7
- Bernoulli random walks, concepts 36
- Bernstein theorem, concepts 53–4
- Bessel functions 49, 71
- binary options *see* digital options
- binomial distribution 27, 36, 160
- bit-reversals, concepts 209–14
- Black–Scholes options pricing model
 - see also* partial differential equations
 - assumptions 5
 - concepts 1–2, 4–5, 23–6, 63–77, 88–93, 129–31, 144–6, 152
 - critique 1–2, 4–5
 - definition 4, 88–9
 - demise 1–2
 - geometric Brownian motion 73–4, 88–93
 - limits 144–6
 - modifications 63–77
 - time-change approaches 63–77, 129–31, 144–6, 152
- Borel sets, concepts 40–1, 155
- bounded support, concepts 15–27, 95–112
- branch cuts/points, concepts 183–4
- Brownian motion
 - see also* diffusion; Lévy processes; random walks
 - characteristic function 34
 - concepts 4–6, 30–4, 41–2, 45–6, 50–3, 88–93, 130–2, 136–41
 - definitions 31, 34
 - semi-martingale processes 6, 64–77
- business time, concepts 6, 57, 63–77

- butterfly spreads, concepts 7–12, 85–93, 98–9
- cadlag processes
 concepts 82–3, 168–70
 definition 168
- calendar time, concepts 6, 57, 63–77
- calibration issues
see also dynamic . . . ; static . . .
 concepts 4–5, 11–12, 23–7, 80–93, 146–52
- call options
 concepts 2–27, 83–93, 121, 127–8, 129–52, 221–3
 put–call parity 84–93
- Carr–Madan approach, concepts 27, 120–2, 129
- Cartesian products 106, 175, 177–8, 205–6
- cash-or-nothing call options
see also digital . . .
 concepts 2–3, 9–10
- cash-or-nothing put options
see also digital . . .
 concepts 2–3, 9–10
- Cauchy distributions, concepts 34, 44–5, 100–2, 166–7, 171, 180–4, 186–90
- Cauchy integral formula, concepts 187–200
- Cauchy–Goursat theorem
 concepts 186–200
 definition 186–7
- Cauchy–Riemann conditions
 concepts 180–4, 187
 definition 180–1
- CDFs *see* cumulative distribution functions
- central limit theorem
see also i.i.d.
 concepts 4–5, 29–55
- CGMY processes
see also variance gamma . . .
 concepts 47–8, 53, 59, 71, 77, 134, 137–40
 definition 47–8
 simulations of Lévy processes 77
 time-change approaches 71, 77, 137–40
- change of measure technique
 concepts 79, 82–93
 definition 82–3
- characteristic exponent
see also Lévy measure
 definition 5–6, 38
- characteristic functions
see also Fourier transform . . . ; Lévy processes
 Brownian motion 34
 compound Poisson processes 40, 46
 concepts 5–12, 14–27, 29, 32–55, 57, 67–77, 126–7, 130–4, 140–6, 160, 166–7
 definitions 5, 6, 9, 21, 32–4, 160, 166–7
 Heston stochastic volatility model 142–6, 228
 positive Poisson point processes 43–4, 61–3
 properties 160
- characteristic integral
 concepts 11–12, 14–27, 129–31
 definition 11, 21–2, 129
- chi-square laws with n degrees of freedom
see also gamma distributions
 concepts 163
- CIR *see* Cox–Ingersoll–Ross process
- CIR stochastic clocks
 concepts 66, 71–2, 142
 definition 66
- circular matrices
 concepts 216–23
 definition 216–17
 Toeplitz matrices 219–20
- class L laws
see also self-decomposable distributions
 concepts 58–9
- clocks
see also time-change . . .
 concepts 6, 57, 64–77
- closed under convergence, definition 97
- closed under that operation, definition 104
- clustering effects of volatility 5–6, 57–63
- commutative operations 107–8
see also convolution
- compact support properties
see also test functions
 concepts 95–112
- complete markets, concepts 3–4, 81–93
- completely monotone Lévy densities 53–4
- completeness factors, Lévy markets 93
- complex conjugate of a complex number,
 definition 177
- complex functions
 concepts 95–112, 116–28, 163–4, 179–84
 definitions 179–80
- complex integration, definitions 185–6
- complex numbers
 concepts 7–12, 173–84, 185–200, 201–6, 208–14
 elementary operations 176–7
 polar form 177–8
 uses 173
- complex planes
 concepts 175–84, 185–200
 definition 175–6
- complex residue, concepts 187–8, 196–9
- complex-valued functions
see also test . . .
 concepts 95–112, 116–28, 163–4, 179–84, 198–200, 202–6
- composite functions 98
- compound Poisson processes
see also Lévy . . .
 characteristic function 40, 46
 concepts 39–41, 44–5, 46, 50–3, 59, 65–6, 74–7, 132–4
 definition 39–40

- simulations 74–7
 subordinators 65–6
 conditional probabilities, concepts 156, 167–8
 conjugate symmetry, concepts 203–4
 continuous dual space, definition 206
 continuous linear functional on the space
 concepts 97–112, 124–8, 205–6
 definition 97–8
 contour complex integration techniques 163
 convergence of sequences of random variables,
 concepts 166–7
 convolution
 concepts 1, 9–12, 21–7, 104–12, 118–28,
 129–52, 207–14, 225–8
 definitions 9–12, 104–12
 direct (tensor) product of distributions 105–6
 distributional convolution 9–12, 27, 105–12,
 127–8, 129–52
 distributions in S 108–12
 function convolution 21, 104–12, 118–28
 Gaussian functions 104–5, 226–7
 properties 104–5
 Cooley–Tukey algorithm
 see also fast Fourier transform
 concepts 208–9
 correlations, concepts 1–3
 cosines 13–14, 113–28
 counting Poisson process, concepts 40–1
 Cox processes
 see also intensity; Poisson . . .
 definition 62–3
 Cox–Ingersoll–Ross process (CIR), concepts 66,
 71–2, 142, 225–7
 crash of 1987 1, 31, 88–9
 cumulative distribution functions (CDFs) 85–93,
 156–7

 daily returns, monthly returns 35
 Danielson–Lanczos algorithm
 see also fast Fourier transform
 concepts 208–9
 data-generating process (DGP), definition 81–2
 DAX 146–52
 De Moivre formula 178
 De Morgan formula 155
 decimal–binary conversion table 209–10
 decomposition theorem, concepts 45–53, 76–7,
 218
 degrees of freedom, dynamic trading strategies 4–5
 derivative of a distribution, definition 100
 derivatives
 see also digital . . . ; exotic . . . ; forward . . . ;
 options . . .
 attainable contracts 3–4, 83–93
 concepts 79, 83–93
 deterministic volatility, Lévy processes 62–3,
 139–41

 DFT *see* discrete Fourier transform
 DGP *see* data-generating process
 differentiability of functions 95, 98–112,
 179–90
 differential calculus 95, 98–112
 diffusion
 see also Brownian motion
 concepts 4–5, 17–26, 31, 45–9, 57, 59–77,
 88–93, 132–6, 138–41
 jump-diffusion processes 59–60, 62–3, 132–6,
 147–52
 digital options
 see also asset-or-nothing . . . ;
 cash-or-nothing . . . ; exotic . . . ; options . . .
 concepts 1–3, 6–12, 27, 84–93, 129–30
 definition 7
 Fourier transform of the payoffs 8–12, 27
 plain vanilla options 86–93
 pricing 1–3, 6–12, 27, 84–93, 129–30
 Dini’s test 115–17
 Dirac delta function
 see also Heaviside . . . ; singular distributions
 concepts 7–12, 85–93, 98–112
 definition 7, 98–9, 100
 direct (tensor) product of distributions
 see also convolution
 concepts 105–6
 Dirichlet conditions 115–17
 discrete Fourier transform (DFT)
 see also Fourier transform
 concepts 207–14, 217–23
 definition 207–8
 uses 207–8
 discrete jump models
 concepts 132–4, 147–52
 market data 147–52
 distribution, probability concepts 95, 105,
 156–69
 distributional convolution
 see also convolution
 concepts 9–12, 27, 105–12, 127–8, 129–52
 distributions
 see also generalized functions
 calculus 99–102
 concepts 95–112, 113, 120–1, 123–8, 160–6
 convolution 9–12, 27, 104–12, 127–8,
 129–52
 definition 95
 examples 100–2
 Fourier transform 1, 6–12, 41–9, 85–93,
 95–112, 113, 120–1, 123–8, 129–52
 slow growth distributions 103–4, 123–8
 Donsker theorem, definition 30–1
 Doob martingale theorems 170–1
 drift, concepts 4–5, 31, 45–9, 65–77, 88–93,
 132–40
 dual space concepts 97–112, 124–8, 205–6

- dynamic trading strategies
 concepts 3–6, 29–55
 definition 4
 non-stationary market dynamics 57–77, 134–40
- efficient market hypothesis (EMH)
 concepts 4–5, 29–49, 79–93
 definition 29
- elementary operations, complex numbers
 176–7
- elements of measure theory, concepts 155–69
- elements of probability, concepts 155–71
- elements of the theory stochastic processes,
 concepts 168–70
- embedded random walks, simulations of Lévy
 processes 74
- EMM *see* equivalent martingale measure
- equity options
see also options . . .
 skew effects 5
- equivalent martingale measure (EMM)
 concepts 3, 82–93
 definition 3, 82
- Erland laws
see also gamma distributions
 concepts 162–3
- Esscher transform, definition 91–3
- Euclidean space 105–6, 204
- Euler’s formula 178
- European options 2–3, 11–27, 83–93, 129–52
see also options . . .
- excess returns
see also returns; Sharpe ratio
 concepts 79–80
- exercise dates 7–12, 84–93, 129–52
- exotic options 73–7, 84–93, 129–30, 146–52
see also Asian . . . ; digital . . . ; options . . .
- expected utility frameworks, concepts 3–4
- expected values, concepts 157–8, 167–8
- exponential distributions
 concepts 36, 162, 166
 definition 162, 166
- factor loading, concepts 81–2
- fast Fourier transform (FFT)
see also Fourier transform
 concepts 14–27, 146–52, 207, 208–14, 215–23
 Cooley–Tukey algorithm 208–9
 critique 26
 Danielson–Lanczos algorithm 208–9
 definition 15–26, 207, 208–10
 FFFT 26, 215–23
 uses 14–27, 207–8, 215–16, 221–3
- FFFT *see* fractional FFT
- FFT *see* fast Fourier transform
- filtrations, definition 168–70
- finite activity jumps
 concepts 5–6, 42–5, 50–3, 132–6, 147–52
 definition 5, 132
 discrete jump model 132–4, 147–52
 Merton jump-diffusion model 133–6, 147–52
- finite variation conditions
 Lévy processes 52–3, 64–77
 stable processes 53
- forward contracts 83–93
- forward Fourier transform, concepts 117–20
- forward prices 6–12
- Fourier cosine transform
 concepts 118–20
 definition 118
- Fourier series
 concepts 113–17
 definition 113–14
 successive approximations of common
 functions 113–14
- Fourier sine transform
 concepts 118–20
 definition 118
- Fourier transform 1, 6–27, 41–9, 57, 79, 85–93,
 95–112, 113–28, 146–52, 160, 207–14,
 215–23
 autocorrelation 119–28
 Brownian motion 34
 Carr–Madan approach 27, 120–2, 129
 common conventions 12–13, 117–18
 concepts 1, 6–27, 57, 79, 85–6, 97–112,
 113–28, 146–52, 160, 207–14, 215–23
 definition 8–12, 15, 21–6, 117–20
 DFT 207–14, 217–23
 digital payoffs 8–12, 27
 distributions 1, 6–12, 41–9, 85–93, 95–112,
 113, 120–1, 123–8, 129–52
 exercises 125–7
 FFFT 26, 215–23
 FFT 14–27, 146–52, 207, 208–14
 a functional concepts 97–8, 129–30
 functions 8–12, 97–8, 113–27
 generalized function approach 1, 6–27, 41–9,
 85–93, 95–112, 113, 120–1, 123–8, 129–52
 IDFT 207–8
 Lewis approach 27, 120, 122–3, 129
 linear properties 118–28
 literature review 26–7
 market data 14–26, 129–30, 146–52
 options pricing 6–12, 14–26, 120–8, 146–52,
 220–3
 overview 1, 14–27
 Poisson processes 36, 40–1
 popularity 1
 real-world pricing applications 14–26, 129–30,
 146–52, 221–3
- fractals, concepts 57–8
- fractional FFT (FFFT)
 concepts 26, 215–23
 numerical results 26, 221–3
- frequencies 113–28, 207–14

- frequency domain representations, concepts 207–14
- Fubini's theorem, concepts 52–3
- function convolution
see also convolution
 concepts 21, 104–12, 118–28
 definition 104–5
- function spaces
 concepts 201–6
 definition 201–2
- functionals concepts 95–112, 129–30, 205–6
- functions, Fourier transform 8–12, 97–8, 113–27
- FX markets, smile effects 4–5
- gamma distributions
 concepts 37, 66–7, 162–6
 definition 162–3, 166
 infinitely divisible distributions 37
- gamma processes 37, 46–7, 53, 59, 65–7, 68–70, 74–7, 134–9, 147–52, 162–6, 221–3
see also Lévy . . .
 concepts 46–7, 53, 65–6, 74–7, 134–9
 finite variation aspects 53
 simulations of Lévy processes 74
 subordinators 65–6
 variance gamma processes 46–7, 53, 59, 68–70, 74–5, 77, 134–9, 147–52, 221–3
- gamma–OU stochastic clocks
 concepts 66–7, 72–3
 definition 66–7
- Gauss, Carl Friedrich 208
see also fast Fourier transform
- Gaussian distributions *see* normal distributions
- Gaussian functions, convolution concepts 104–5, 226–7
- general equilibrium models, concepts 79–93
- generalized functions
see also test . . . ; vector spaces
 calculus of distributions 99–102
 concepts 1, 6–27, 41–9, 85–93, 95–112, 113, 120–1, 123–8, 129–52
 convolution 9–12, 27, 104–12, 127–8, 129–52
 definition 7, 95
 slow growth distributions 103–4, 123–8
- generalized hyperbolic processes, definition 49
- geometric Brownian motion
 Black–Scholes options pricing model 73–4, 88–93
 concepts 4–5, 31, 73–7
 definition 31
- geometric distributions, infinitely divisible distributions 37
- Green theorem, concepts 186–7
- harmonic analysis
see also Fourier series
 concepts 113–17
- hat notation convention for the Fourier transform 12–13
- Heaviside function
see also Dirac delta . . .
 concepts 6–12, 84–93, 100–12
 definition 6–7
- heavy tails 34, 47–9, 54–5
- hedging errors, definition 93
- Heston stochastic volatility model
see also stochastic volatility
 characteristic function 142–6, 228
 concepts 19–20, 71–2, 141–6, 147–52, 222–3, 225–8
 definition 71–2, 141–2
 exotic options 147–52
 options pricing 142–6, 147–52, 222–3
 plain vanilla options 142–6
- Hilbert transform
 concepts 12–15, 21, 129–30, 205
 definition 12–13
- IDFT *see* inverse discrete Fourier transform
- idiosyncratic risk, concepts 81–93
- i.i.d. 30–1, 62–3, 132, 160–6
see also central limit theorem
- imaginary numbers
see also complex numbers
 concepts 174–84
 definition 174–5
- implied volatilities 4–5, 132–52
see also smiles
- in-the-money options 4–5, 83–93
- incomplete markets, concepts 3–4, 93
- independent increments, concepts 4–6, 29–55, 57–77, 90–3
- index of random variables, concepts 33–4
- infinite activity jumps
see also CGMY processes; variance gamma processes
 concepts 5–6, 43–5, 134–40, 147–52
 definition 5
- infinite divisibility
see also self-decomposable distributions
 concepts 30–1, 35–9, 48–9, 54–5, 57, 59–77
 definition 30, 36–7
 distribution types 37, 48–9, 65–77
 non-stationary market dynamics 57, 59–77
- infinite summation 99–100
- infinitely smooth functions
see also test . . .
 concepts 95–112
- information
 arrival-of-information probability laws 6, 39–49, 57
 efficient market hypothesis 4–5, 29–49, 79–93
 Lévy markets 39–49

- inner product space
 concepts 203–6
 definition 203–4
- innovations
 concepts 29–55
 random walk model 29–30
- insider information 29–30
- instantaneous (business) activity rate, concepts 66
- integers, concepts 173
- integration, concepts 98–112, 113–28, 157–60
- intensity
see also Cox processes
 concepts 5–6, 40–6, 50–3, 62–3, 90–3
- interest rate models 66, 71–2, 142, 225–7
- interest rate options, smile effects 4–5
- inverse discrete Fourier transform (IDFT)
see also Fourier transform
 concepts 207–8
 definition 207
- inverse Fourier transform
see also Fourier transform
 concepts 15–27, 117–20, 122–8
- inverse Gaussian distributions
 concepts 34, 65–6, 69–71, 77
 subordinators 65–6
- isotropic derivatives, concepts 180
- Jacobian determinants 106–7
- joint dynamics 5–6
- Jordan lemma, concepts 199–200
- jump-diffusion processes, concepts 59–60, 62–3, 132–6, 147–52
- jumps
see also finite activity . . . ; infinite activity . . . ; Poisson processes
 concepts 5–6, 36–45, 50–3, 57–77, 90–3, 129–52, 168–70
 discrete jump model 132–4, 147–52
 Merton jump-diffusion model 133–6, 147–52
- Khintchine theorem, definition 35–6
- Kronecker delta 98–9, 113–14, 208
- kurtosis 47–8, 54–5, 57–63, 137–9, 158
- lack of memory property, definition 162
- Laplace transformations 97, 181–2
see also Fourier . . .
- Laurent series, concepts 193–200
- Lebesgue integrals 97–8, 155–6, 158–9
- leptokurtosis 55
- leverage effect, concepts 66
- Lévy markets
 arbitrage-free pricing 92–3
 completeness factors 93
 construction 39–49, 92–3
 definition 92
- Lévy measure
see also characteristic exponent
 CGMY processes 47–8
 concepts 5–6, 38, 47–8, 57–77
 definition 5, 38
- Lévy processes
see also Brownian motion; CGMY . . . ; gamma . . . ; Markov . . . ; Meixner . . . ; Poisson . . . ; stable . . . ; variance gamma . . .
 additive processes 60–77
 arrival-of-information probability laws 6, 39–49, 57
 characteristics 45–9, 52–5
 completely monotone Lévy densities 53–4
 compound Poisson processes 40–1, 44, 46, 50–3, 59, 65–6, 74–7
 concepts 5–6, 29–55, 57–77, 88–93
 definitions 5, 30, 32, 35–6, 45
 deterministic volatility 62–3, 139–41
 finite variation conditions 52–3, 64–77
 list of processes 46–9, 59
 martingale processes 89–93
 moments 54–5
 pathwise properties 49–53
 properties 49–55
 random walks 30–1, 74–7
 self-similar processes 58
 simulations 73–7
 subordinators 64–77
 total variation of Lévy processes trajectories 50–3
- Lévy–Itô decomposition theorem, concepts 45–8, 49–53, 62–3
- Lévy–Khintchine representation
 concepts 5–6, 29, 37–55, 57, 60–77, 90–3
 definition 5, 38, 47
- Lévy–Khintchine theorem
 concepts 5, 37–8, 44–5
 definition 5, 37–8, 47
- Lewis, A.L. 27, 120, 122–3, 129
- Lindeberg–Levy theorem, definition 30
- linear properties, Fourier transform 118–28
- Liouville theorem, concepts 190
- liquidity, time-change approaches 57
- literature review, Fourier transform 26–7
- locally finite measures, concepts 155–6
- locally integrable functions, regular distributions 98–112
- log-normal distributions, concepts 4–5, 15–16, 31, 133–6
- long positions 2–3
- market data
 calibration issues 4–5, 11–12, 14–27, 80–93, 146–52
 Fourier transform 14–26, 129–30, 146–52

- market price of risk
 concepts 81, 88–93
 definition 81
- Markov processes
see also additive . . . ; Lévy . . .
 concepts 4–5, 35–6, 60–3
- Markovian prices, concepts 4–5
- martingale pricing theory, definition 81–2
- martingales
 concepts 3, 6, 44–5, 80–93, 121–2, 127–8, 132, 137–40, 170–1
 definition 82, 170
 Doob martingale theorems 170
 Lévy processes 89–93
- matrix vector multiplication, circular matrices 218
- mean
 efficient market hypothesis 29–30
 mean-variance optimizations 3–4
- measure theory, elements 155–69
- Meixner processes
see also Lévy . . .
 concepts 48–9, 59, 71
 definition 48–9
 time-change approaches 71
- memory property, definition 162
- Merton jump-diffusion model
 concepts 133–6, 147–52
 market data 147–52
- model mis-specification risks, dynamic trading strategies 4–5
- moments, concepts 54–5, 157–8
- moneyness of the options, concepts 4–5, 6–12, 83–93, 129–52
- monthly returns, daily returns 35
- movie analogy 1, 129–30
- multi-valued functions
see also complex . . .
 concepts 181–4
- multiplication of functions, concepts 98
- natural numbers
 concepts 173–84
 definition 173
- NIG processes
 concepts 69–71
 definition 69–70
 time-change approaches 69–71
- no-arbitrage conditions, concepts 1–5, 79–93
- non-stationary market dynamics
 concepts 5–6, 57–77, 134–40
 infinite divisibility approach 57, 59–77
 self-decomposable distributions 57–63
 self-similar processes 57–63
 simulation of Lévy processes 73–7
 subordination technique 67–77
 time-change approaches 5–6, 57, 63–77, 134–40
- normal distributions
 concepts 1–3, 4–5, 31, 133–6, 161, 166
 critique 1–3, 31
 definition 161, 166
 infinitely divisible distributions 37, 38
- normalization 127, 207–8
- odd functions 101–2, 115–17
- open sets 205
- options pricing
see also call . . . ; digital . . . ; European . . . ; Fourier transform; pricing; put . . .
 Asian options 146–52
 Black–Scholes options pricing model 1–2, 4–5, 23–6, 63–77, 88–93, 129–31, 144–6, 152
 Carr–Madan approach 27, 120–2, 129
 concepts 1–27, 59–60, 79–93, 100–2, 120–8, 129–52, 220–3
 European options general formula 11–12
 exotic options 73–7, 84–93, 129–30, 146–52
 general representation 1–3, 129–30
 Heston stochastic volatility model 142–6, 147–52, 222–3
 Lewis approach 27, 120, 122–3, 129
 real-world pricing applications 14–26, 129–30, 146–52, 221–3
 Toeplitz matrices 220
- ordinary differential equations 113–28
- oscillation frequencies 113–28
- out-of-the-money options 4–5
- parameters
 dynamic trading strategies 4–5
 Lévy processes 46–9
- Parseval theorem
 concepts 120, 123–4
 definition 120
- partial differential equations (PDEs) 88–93, 181–4, 207–14, 227
see also Black–Scholes options pricing model
- path integral approach, concepts 225–8
- pathwise properties of Lévy processes 49–53
- payoff generalized function, concepts 1, 6–27, 85–93, 98–9, 122–3, 127–8, 129–52
- payoffs
 concepts 1, 6–27, 83–93, 98–9, 122–3, 127–8, 129–52, 221–3
 definition 6
- PDEs *see* partial differential equations
- plain vanilla options
see also options . . .
 digital options 86–93
 Heston stochastic volatility model 142–6
- Poisson distributions
 concepts 37–8, 161, 166
 definition 161, 166
 infinitely divisible distributions 37–8

- Poisson point process
 concepts 41–6, 61–3, 74–7
 definition 41–2
 simulations of Lévy processes 74–7
 sums over Poisson point processes 42–5
- Poisson processes
see also Cox . . . ; jump . . . ; Lévy . . .
 arrival-of-information probability laws 39–45
 compound Poisson processes 39–41, 44, 46,
 50–3, 59, 65–6, 74–7, 132–4
 concepts 36–8, 39–51, 61–3, 65–6, 90–3,
 132–4, 161
 definitions 36, 39–40, 41–2, 46
 Fourier transform 36, 40–1
 Lévy markets 39–45
 subordinators 65–6
 sums over Poisson point processes 42–5
 thinning properties 41
- polar form of complex numbers, concepts
 177–8
- police story analogy 1, 129
- price discovery processes, concepts 29–30
- pricing
see also Fourier transform . . . ; options pricing
 arbitrage-free pricing 79–93, 129–52
 arrival-of-information probability laws 6,
 39–49, 57
 Asian options 146–52
 Black–Scholes options pricing model 1–2, 4–5,
 23–6, 63–77, 88–93, 129–31, 144–6, 152
 Carr–Madan approach 27, 120–2, 129
 change of measure technique 79, 82–93
 concepts 1–27, 59–60, 73–7, 79–93, 98–102,
 120–8, 129–52, 220–3
 digital options 1–3, 6–12, 27, 84–93, 129–30
 dynamics 3–6, 29–55
 European options general formula 11–12
 examples of distributions 190–2
 exotic options 73–7, 84–93, 129–30, 146–52
 general representation 1–3, 129–30
 Lewis approach 27, 120, 122–3, 129
 real-world pricing applications 14–26, 129–30,
 146–52, 221–3
 Toeplitz matrices 220
- pricing kernels
 concepts 1, 6–27, 86–93, 129–52
 definition 6, 86, 129
- principal value integrals
 concepts 190–3
 definition 190–1
- probability
 concepts 79, 85–93, 95, 105, 112, 155–71
 elements 155–71
- probability density functions (PDFs) 15–26,
 85–93, 112, 225–8
- process with small jumps thrown away 76–7
- put options, concepts 2–27, 84–93, 129–52, 221–3
- put–call parity
 concepts 84–93
 definition 84
- Python 178
- Radon measures, concepts 155–6
- Radon–Nikodym derivatives
 concepts 10–12, 81–3, 90–3, 167
 definition 82–3, 167
- random walks
see also Brownian motion; shocks; stationary
 independent increments
 concepts 29–30, 40–1, 74–7
 definition 29, 30
 embedded random walks 74
 Lévy processes 30–1, 74–7
 simulations of Lévy processes 74–7
- rapid descent functions
see also test functions
 concepts 103–4, 109–12, 123
 definition 103
- rational expectations theory 79–93
- rational numbers
 concepts 32–4, 37, 173–84
 definition 173
- real numbers 32–4, 105–12, 116–28, 159, 173–84,
 201–6, 215–23
see also complex numbers
- real-valued random variables 156–7
- real-world pricing applications, Fourier transform
 14–26, 129–30, 146–52, 221–3
- reflection operator 127–8
- regular distributions
 concepts 98–112
 definition 98
- replicating portfolio technique
 concepts 83–93
 definition 83–4
- residue theorem, concepts 187–8, 196–9
- returns
see also excess returns
 daily/monthly returns 35
 risk 79–93
- Riccati equations, concepts 227–8
- Riemann integrals, concepts 158–9, 180–3, 187
- risk 1–5, 29–30, 79–93, 121–2, 127–8, 132–3,
 138–41, 225–8
 averse investors 79–80
 management concepts 1–3
 market price of risk 81, 88–93
 returns 79–93
- risk premiums 29–30, 79–93
 concepts 79–93
 efficient market hypothesis 29–30, 79–93
- risk-free discount factors 2–3
- risk-free rates, concepts 3–5, 80–93
- risk-less assets 79–93

- risk-neutral probabilities
 concepts 1–5, 80, 82–93, 121–2, 127–8, 132–3, 138–41, 225–8
 derivation 1–2
- risky assets, arbitrage-free pricing 79–93
- sampling theorem
 concepts 15–26
 critique 21
 definition 15–17
 truncated sampling theorem 17–26
- Sato processes
see also self-decomposable distributions
 definition 63
- scaling property, concepts 34, 39, 99, 127
- self-decomposable distributions
see also infinite divisibility
 concepts 57–63
 definitions 58–9
 Sato processes 63
- self-similar processes
see also stochastic . . . ; volatility
 concepts 57–63
 definition 57–8
 Lévy processes 58
- semi-martingale processes, Brownian motion 6, 64–77
- semi-strong efficient markets
 concepts 29–30
 definition 29
- Sharpe ratio
see also excess returns; volatility
 concepts 79–80
 definition 79
- shifting property, concepts 99, 120
- shocks
see also random walks
 arrival-of-information probability laws 6, 39–49, 57
 concepts 4–5, 29–55
- short positions 2–3
- signal processing
see also unit impulse functions
 concepts 98–9, 207–8
- sines 13–17, 113–28
- singular distributions
see also Dirac delta . . .
 concepts 27, 98–112
 definition 98–9
- skew effects
 concepts 1–3, 5, 33–4, 47–9, 54–5, 57–63, 65–6, 137–9, 158
 definition 5
- Skorohod theorem, definition 35–6
- slow growth distributions
see also tempered distributions
 concepts 103–4, 123–8
- definition 103
- smiles
see also implied volatilities; strike prices
 concepts 4–5, 11–12, 129–30, 132–45
 definition 4
- smooth functions, concepts 22–3, 95–112, 179–80
- speculation, ‘The theory of speculation’ (Bachelier) 29
- square integrable martingales, definition 170
- square roots 142–6, 173–84
- stable distributions
 concepts 31–5, 38–9, 57–63, 77
 definitions 31–2
- stable Lévy processes
 concepts 32, 46, 58–63, 77
 definition 32
- stable processes
 concepts 31–4, 38–9, 46–7, 53, 57–63, 65–6, 77
 finite variation conditions 53
- stable subordinators, concepts 65–6
- static replication approaches
 concepts 4, 83–4
 definition 4
- stationarity of the increments of log-prices, concepts 5–6, 55, 57–77
- stationary independent increments
see also Lévy processes; random walks
 concepts 5–6, 29–55, 57–77, 90–3
 critique 5–6, 55, 57
 definitions 35
- stochastic clocks
see also subordinators; time-change . . .
 concepts 6, 57, 64–77
 definition 64
- stochastic differential equations 31
- stochastic processes 5–6, 39–49, 57–77, 82–93, 168–70, 225–8
see also Brownian motion; Lévy . . . ; Poisson . . . ; self-similar . . .
 additive processes 60–77
 definition 168–70
 elements of the theory 168–70
- stochastic volatility
see also Heston . . .
 concepts 19–20, 66–7, 130, 138–46, 147–52, 222–3, 225–8
 definition 66, 138–41
- stopped processes, concepts 170–1
- stopping times, definition 170–1
- strike prices
see also smiles
 concepts 2–12, 27, 83–93, 98–9, 129–52, 220–3
- strongly efficient markets
 concepts 29–30
 definition 29

- subordination technique
see also time-change approaches
 concepts 67–77
- subordinators
see also stochastic clocks
 building examples 65–6
 concepts 6, 57, 64–77
 definition 64–5
- sums over Poisson point processes, concepts 42–5
- superposition principle 113–28
- symmetric stable distributions, concepts 34, 77
- Taylor expansion 43–4, 195–6
- tempered distributions
see also slow growth distributions
 concepts 103–4
- term structures of volatility, concepts 5, 11–12, 57–63
- term-by-term transformations 125
- test functions
see also complex-valued . . . ; generalized . . . ;
 rapid descent . . . ; vector spaces
 concepts 7–12, 95–112, 123–8
 definition 7, 95–7, 103
 direct (tensor) product of distributions 105–6
- thinning properties of Poisson processes, concepts 41
- time discretization, concepts 74
- time-change approaches
 Barndorff-Nielsen–Shephard model 72–3
 CGMY processes 71, 77, 137–40
 characteristic functions 5–6
 concepts 5–6, 57, 63–77, 134–46
 Heston stochastic volatility model 71–2, 141–6, 147–52
 Meixner processes 71
 NIG processes 69–71
 non-stationary market dynamics 5–6, 57, 63–77, 134–40
 variance gamma processes 68–70, 74–5, 77, 134–9, 147–52
- time-changed Lévy processes, concepts 63
- time-delayed Dirac delta, concepts 99
- time-dependent volatility case, additive processes 62–3
- Toeplitz matrices
 circular matrices 219–20
 concepts 216, 219–23
 definition 216, 219–20
 pricing 220
 uses 220
- topological vector spaces, concepts 205
- total variation of Lévy processes trajectories, concepts 50–3
- triangular arrays, definition 35–6
- trigonometric functions 13–14
- truncated Poisson point processes, simulations of Lévy processes 74–7
- truncated sampling theorem, concepts 17–26
- TV *see* total variation . . .
- underlying assets 1–27, 79–93
- unit impulse functions
see also signal processing
 concepts 98–9
- variance, concepts 34–5, 41–2, 46–7
- variance gamma processes
see also CGMY . . . ; Lévy . . .
 concepts 46–7, 53, 59, 68–70, 74–5, 77, 134–9, 147–52, 221–3
 definition 46–7
 finite variation aspects 53
 market data 147–52
 simulations of Lévy processes 74–5, 77
 time-change approaches 68–70, 74–5, 77, 134–9, 147–52
- vector spaces
see also generalized functions; test functions
 concepts 95–112, 124–8, 201–6
 definition 95–7, 201–2
 topological vector spaces 205
- volatility
see also Sharpe ratio; skew . . . ; smile . . . ;
 stochastic . . .
 clustering effects 5–6, 57–63
 concepts 1–3, 4–5, 11–12, 17–26, 57–77, 79–93, 132–52
 self-similar processes 57–63
 term structures of volatility 5, 11–12, 57–63
- weakly efficient markets, definition 29–30
- Wiener process
see also Brownian motion
 concepts 4–5
- Wiener–Khintchine theorem, definition 119–20
- Zemanian theorems 106–8, 123–4
- zero forecasts, concepts 4
- Index compiled by Terry Halliday*