1
Main Strategic Equity Derivative Instruments

This chapter provides a good understanding of the equity derivative instruments most widely used by equity derivatives professionals. This is the most complex and technical chapter of this book. It aims to solidify the reader's technical knowledge of these instruments. I have also tried to emphasize the practical aspects of these instruments when applied to strategic equity transactions. I start with a discussion of less complex instruments such as equity forwards and equity swaps. I continue covering stock lending transactions – although not derivative instruments, they nonetheless are a key component of strategic equity transactions. Options are addressed next, starting with the basics and progressing to an explanation of option sensitivities and delta-hedging. Finally, I include more specialized equity derivative products such as dividend swaps, variance swaps and volatility swaps.

1.1 EQUITY FORWARDS

1.1.1 Equity Forwards

Equity forwards allow an investor to take bullish or bearish views on an underlying stock, a basket of stocks or a stock index.

- A **physically settled equity forward** is an agreement between two counterparties whereby one counterparty – the buyer – agrees to buy from the other counterparty – the seller – a specified number of shares of a specified stock or basket of stocks, at a specified time in the future – the settlement date – at a pre-agreed price – the forward price. This instrument is called a physically settled forward, because the underlying shares are delivered by the seller to the buyer. The buyer and the seller pay no upfront premium to enter into the equity forward.
- A **cash-settled equity forward** is an agreement between two counterparties whereby one counterparty – the buyer – receives at a specified time in the future – the settlement date – from the other counterparty – the seller – the appreciation of the underlying stock, basket of stocks or stock index, above a pre-agreed price – the forward price. Conversely, the seller receives from the buyer the depreciation of the underlying below the forward price. This forward is called a cash-settled forward, because no underlying shares are delivered to the buyer at maturity. Only cash is paid by one party to the other. The buyer and the seller pay no upfront premium to enter into the equity forward.
- Often a forward can be both cash-settled and physically settled, giving one of the two counterparties the right to choose the type of settlement just prior to maturity.

An equity forward agreement is formalized through a confirmation. The confirmation is generally legally subject to the terms and clauses of the International Swaps and Derivatives

Association (ISDA) Master Agreement signed between the two counterparties. As its name suggests, once signed, the Master Agreement governs all past and future individual derivative transactions entered into between the two counterparties.

1.1.2 Example of a Cash-settled Equity Forward on a Stock

Let's assume that our entity ABC Corp. has a positive view on Deutsche Telekom (DTE) stock for the next three months. As a result, ABC is considering entering into an equity forward. As seen earlier, a forward can be either physically settled or cash-settled. In a physically settled equity forward the buyer will pay to the seller an amount equal to the forward price multiplied by the number of shares, and the seller will deliver to the buyer the number of shares. In a cash-settled forward the appreciation or depreciation of the shares relative to the forward price is exchanged between the two counterparties. Because ABC is not interested in receiving DTE shares at maturity, ABC enters into a 3-month cash-settled equity forward on 10 million shares of DTE, with the following terms:

Equity Forward Main Terms	
Buyer	ABC Corp.
Seller	Gigabank
Trade date	20-September-20X1
Shares	Deutsche Telekom
Number of shares	10 million
Forward price	EUR 15.00
Settlement price	The closing price of the shares on the valuation date
Valuation date	20-December-20X1
Exchange	Eurex
Settlement method	Cash settlement
Cash settlement amount	The absolute value of: Number of shares $\times$ (Settlement price $-$ Forward price) With the convention that: If Settlement price $>$ Forward price, the amount shall be paid by the seller If Settlement price $<$ Forward price, the amount shall be paid by the buyer
Settlement date	23-December-20X1 (three exchange business days after the valuation date)

During the life of the equity forward, the flows between the two counterparties are the following.

At inception, the forward agreement is signed by the counterparties. No flows take place, as the buyer and the seller pay no upfront premium to enter into the equity forward.

Until maturity of the forward, no flows take place.

At maturity, the "settlement price" will be calculated on the "valuation date". In this example, the settlement price is the closing price of DTE stock on 20 December 20X1 (i.e., the valuation date). Immediately after, the "cash settlement amount" will be calculated as the absolute value

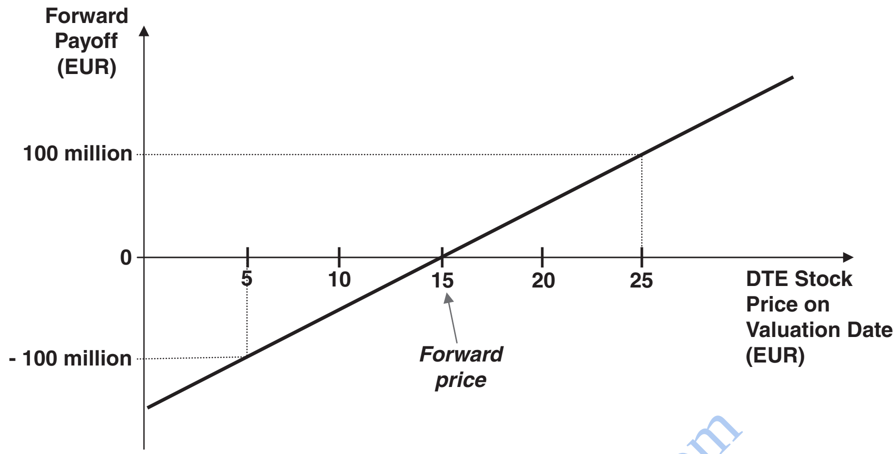

Figure 1.1 Equity forward payoff at maturity.

of the product of (i) the "number of shares" and (ii) the difference between the settlement price and the forward price. Three potential scenarios may take place:

- If DTE stock has appreciated relative to the forward price (i.e., the settlement price is greater than the forward price), ABC would receive from Gigabank the stock appreciation times the number of shares (i.e., the cash settlement amount) on the settlement date.
- Conversely, if DTE stock has depreciated relative to the forward price (i.e., the settlement price is lower than the forward price), ABC would pay to Gigabank the stock depreciation times the number of shares (i.e., the cash settlement amount) on the settlement date.
- If DTE stock has ended up at the same level as the forward price (i.e., the settlement price is equal to the forward price), no cash flows take place.

Let us assume that on the valuation date (20 December 20X1), DTE stock closes at EUR 18.00. The settlement price would then be EUR 18.00. ABC would receive from Gigabank EUR 30 million [= 10 million shares × (18.00 – 15.00)] on the settlement date (23 December 20X1).

Let us assume instead that on the valuation date (20 December 20X1), DTE stock closes at EUR 13.00. The settlement price would then be EUR 13.00. ABC would pay to Gigabank EUR 20 million [= 10 million shares × (15.00 – 13.00)] on the settlement date (23 December 20X1).

Figure 1.1 shows the profit or loss to ABC as a function of DTE's stock price at maturity. Therefore, DTE's maximum profit is unlimited while its maximum loss is limited to EUR 150 million (= 10 million shares × 15.00) reached if DTE's stock price is zero at maturity.

1.1.3 Example of a Physically Settled Equity Forward on a Stock

Let us assume that our entity, ABC Corp., plans to acquire 10 million shares of Deutsche Telekom (DTE) in three months' time. ABC is worried that DTE's stock price might increase

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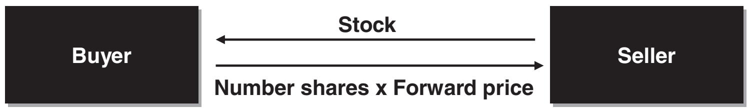

Figure 1.2 Physically settled equity forward, settlement at maturity.

during the next three months. As a result, ABC enters into a physically settled forward on 10 million shares of DTE, with the following terms:

Equity Forward Main Terms	
Buyer	ABC Corp.
Seller	Gigabank
Trade date	20-September-20X1
Shares	Deutsche Telekom
Number of shares	10 million
Forward price	EUR 15.00
Exchange	Eurex
Settlement method	Physical settlement
Settlement date	23-December-20X1

During the life of the equity forward, the flows between the two counterparties are the following.

At inception, the forward agreement is signed by the counterparties. No flows take place, as the buyer and the seller pay no upfront premium to enter into the equity forward.

Until maturity of the forward, no flows take place.

At maturity, "on the settlement date", the buyer – ABC – will pay to the seller – Gigabank – an amount equal to the forward price multiplied by the number of shares, and the seller – Gigabank – will deliver to the buyer – ABC – the number of shares of DTE, as shown in Figure 1.2. In other words, ABC will pay EUR 150 million (= 10 million × 15.00) in exchange for 10 million shares of DTE.

1.1.4 Calculating the Forward Price of a Stock

The easiest way to calculate a forward price of a stock is to come up with a riskless strategy, like the following:

- At inception, we buy one share of the stock paying the then prevailing stock price (i.e., the spot price). This purchase is financed at an interest rate. The overall cash flow at inception is zero, as the financing amount is invested in the stock.
- Also at inception, we will enter into a forward to sell the shares on a specific date (the maturity date).
- During the life of the forward, we will be lending the shares and receiving a fee, called the borrowing fee. We will invest any borrowing fee received until maturity at the then prevailing interest rate.
- During the life of the forward, we will be receiving the dividends distributed to the share. In our case the dividend would be paid to us by the stock borrower. We will invest the dividends until maturity.

- At maturity, we will sell the shares through the forward receiving the forward price, repay the financing, receive the amount resulting from the investment of the received dividends and receive the amount resulting from the investment of the received borrowing fee.

The cash flows at maturity are as follows:

$$\text{Forward price} - \text{Financing repayment amount} + \text{Reinvested dividends} \\ + \text{Reinvested borrowing fee}$$

This cash flow has to be zero if no arbitrage opportunities are present (i.e., if the forward was priced accordingly). Therefore:

$$\text{Forward price} - \text{Financing repayment amount} + \text{Reinvested dividends} \\ + \text{Reinvested borrowing fee} = 0$$

The amounts due to the reinvestment of the dividends and the fee are equivalent to their future value to maturity (FV). Rearranging the terms:

$$\text{Forward price} = \text{Financing repayment amount} - \text{FV(dividends)} - \text{FV(borrowing fee)}$$

Therefore, dividends paid out by the underlying stock, which lower the stock price on the ex-dividend date, have a negative effect on the value of the forward. The borrowing fee received for lending the stock also has a negative effect on the value of the forward.

The financing repayment amount can be expressed as the sum of (i) the spot price and (ii) the interest rate carry. As a result:

$$\textbf{Forward} = \textbf{Spot} + \textbf{Interest rate carry} - \textbf{FV(dividends)} - \textbf{FV(borrowing fee)}$$

If the forward matures in less than a year, the interest carry is calculated as:

$$\textbf{Interest carry} = \textbf{Spot} \times \textbf{Interest rate} \times \textbf{Day count fraction}$$

The following table summarizes the impact on the forward price of an increase in the specified variable, assuming everything else is equal:

	Forward price
Spot price	↑
Interest rate	↑
Dividends	↓
Borrowing fee	↓

As an example, let us assume that a stock is trading at EUR 100. The 3-month interest rate is 5% Actual/360. The company pays a EUR 2 dividend in one month (i.e., in 31 days). The borrowing fee is 0.20% annual Actual/365. The forward interest rate starting in one month and with a 2-month maturity is 4% annual Actual/360. The theoretical 3-month forward, assuming 92 calendar days in the period, is calculated as follows:

$$\begin{aligned}\text{Forward} = {} & 100 + [100 \times 5\% \times 92/360] - [2 \times (1 + 4\% \times 62/360)] \\ & - [100 \times 0.20\% \times 92/365]\end{aligned}$$

Thus, the forward price is 99.21.

1.2 EQUITY SWAPS

Equity swaps are a convenient way to gain either long or short exposure to an equity underlying. The underlying can be a stock, a basket of stocks or a stock index. Based on the type of settlement, an equity swap can be either:

- A **cash-settled equity swap** – an agreement between two counterparties where one party receives the appreciation (and pays the depreciation) of a stock, a basket of stocks or a stock index in exchange for the payment of a stream of interest flows and sometimes the receipt of dividend payments. The other party has the opposite position.
- **A physically settled equity swap** – an agreement between two counterparties whereby one counterparty agrees to buy from the other counterparty the underlying shares and pays a pre-agreed amount in exchange for the payment of a stream of interest flows and sometimes the receipt of dividend payments.

An equity swap is an over-the-counter transaction between two counterparties. It is formalized through a confirmation. The confirmation is generally legally subject to the terms and clauses of the ISDA agreement signed between the two counterparties that sets out their obligations. Hereafter, I will be using the terms as set out by ISDA to cover the mechanics of an equity swap.

1.2.1 Total Return Equity Swaps

A **total return equity swap** is a transaction in which one party –the equity swap receiver – has a position equivalent to a long position in a stock, a basket of stocks or an equity index, while the other party – the equity swap payer – has the opposite position in the same underlying. In a total return equity swap, the equity amount payer and the equity amount receiver exchange during the life of the equity swap three strings of cash flows (see Figure 1.3):

- The **equity amount** reflects the price performance of a long position in the underlying stock relative to its initial price – the **reference price**. The **equity amount receiver** is the counterparty that benefits if the stock performance is positive. Conversely, the **equity amount payer** is the counterparty that benefits if the stock performance is negative.
- The **floating amount** reflects the cost of carrying the underlying stock. It is paid by the equity amount receiver to the equity amount payer. The floating amount is quoted as a floating interest rate plus a fixed spread. The floating rate is typically Libor, Euribor or a similar benchmark rate. The fixed spread is set at swap inception. Typically, the floating amount payments are made every three months, based on the equity swap notional.
- The **dividend amount** reflects the benefits of carrying the underlying stock. It is paid by the equity amount payer to the equity amount receiver.

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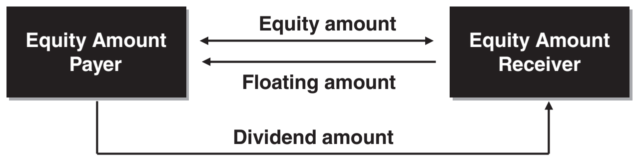

Figure 1.3 Total return equity swap cash flows.

The equity swap is called a total return equity swap because the equity amount receiver receives not only the appreciation of the stock price relative to the reference price but also the dividends distributed to the underlying stock. The equity amount receiver also pays the floating amount, which in a way resembles the interest payments of financing an investment with a notional equal to the equity amount. Therefore, the equity swap mimics a fully financed long position in the stock.

1.2.2 Price Return Equity Swaps

A **price return equity swap** is a transaction in which one party – the equity swap receiver – has a position equivalent to a long position in only the **price** of a stock, a basket of stocks or an equity index, while the other party – the equity swap payer – has the opposite position in the same underlying. In a price return equity swap, the equity amount payer and the equity amount receiver exchange during the life of the equity swap two strings of cash flows (see Figure 1.4):

- The **equity amount** reflects the price performance of a long position in the underlying stock relative to its initial price – the **reference price**. The appreciation of the stock price is received by the **equity amount receiver** from the **equity amount payer**. If, on the other hand, the stock depreciates, then the absolute value of the depreciation is paid to the equity amount payer from the equity amount receiver.
- The **floating amount** reflects the cost of carrying the underlying stock. It is paid by the equity amount receiver to the equity amount payer.

The equity swap is called a price return equity swap because the equity amount receiver receives only the appreciation of the stock relative to the reference price. The equity amount receiver does not receive the dividends distributed to the underlying stock. Therefore, the equity swap does not mimic a long position in the stock. On a stock that pays no dividends, a total return equity swap and a price return equity swap are the same instrument.

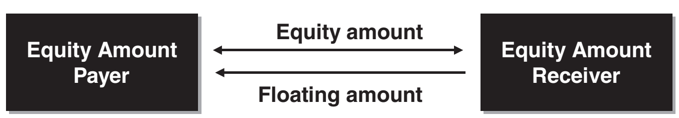

Figure 1.4 Price return equity swap cash flows.

1.2.3 Case Study: Physically Settled Total Return Equity Swap on Deutsche Telekom

In this subsection, I will cover in detail the mechanics of a physically settled total return equity swap on a stock with an example. Let us assume that ABC Corp. has 10 million shares of Deutsche Telekom (DTE), worth EUR 140 million. ABC wants to raise financing while maintaining economic exposure to the DTE stock. To raise the financing, ABC agrees with Gigabank to execute a sale of the DTE stake and to simultaneously enter into a physically settled total return equity swap.

Flows of the Transaction

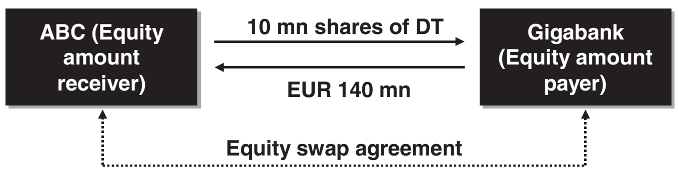

Figure 1.5 Flows on trade date.

The flows of the transaction on the trade date are as follows (see Figure 1.5):

- ABC sells its stake in DTE to Gigabank at the then prevailing stock price of EUR 14 per share. Thus, ABC raises EUR 140 million. Normally, the sale is executed through the stock exchange (i.e., ABC does not face Gigabank directly in the sale).
- ABC and Gigabank enter into a physically settled total return equity swap. The equity swap obliges ABC to buy back its DTE stake in one year's time at the same EUR 14 price per share.

The transaction flows during the life of the equity swap are as follows (see Figure 1.6):

- ABC pays an interest, the floating amount, to Gigabank based on the EUR 140 million equity swap notional.
- Gigabank pays to ABC an amount, the dividend amount, equivalent to the dividends distributed to the underlying DTE shares. This amount is paid on the same date the dividends are paid by DTE to its shareholders.

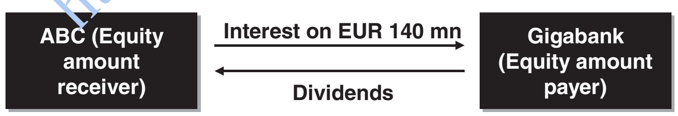

Figure 1.6 Flows during the life of the equity swap.

The transaction flows at maturity (i.e., on the settlement date of the equity swap) are as follows (see Figure 1.7):

- Through the physical settlement of the equity swap, ABC buys back its stake in DTE from Gigabank at a price of EUR 14 per share. ABC then pays EUR 140 million to Gigabank in exchange for 10 million shares of DTE.

It can be seen that all these flows are equivalent to Gigabank providing a EUR 140 million loan to ABC collateralized by 10 million shares of DTE. In theory, Gigabank should be offering better financing terms to ABC than in a standard loan because in case of ABC becoming insolvent, Gigabank can sell the DTE stock in the market and partially or totally repay the loan.

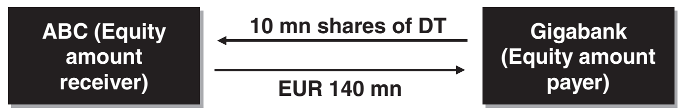

Figure 1.7 Flows on the settlement date of the equity swap.

Main Terms of the Equity Swap

The main terms of the equity swap are shown in the following table:

Physically Settled Total Return Equity Swap Terms	
Party A	ABC Corp.
Party B	Gigabank
Trade date	12-December-20X1
Effective date	15-December-20X1
Termination date	15-December-20X2
Underlying currency	EUR
Equity amount part	
Equity amount payer	Party B (Gigabank)
Equity amount receiver	Party A (ABC Corp.)
Shares	Deutsche Telekom common shares
Calculation agent	Party B (Gigabank)
Physical settlement	Applicable
Number of shares	10 million
Initial price	EUR 14.00
Equity notional amount	EUR 140 million Number of shares × Initial price
Settlement date	15-December-20X2
Floating amount part	
Floating amount payer	Party A (ABC Corp.)
Notional amount	The equity notional amount
Payment dates	At the end of each quarter (15-Mar-20X2, 15-Jun-20X2, 15-Sep-20X2 and 15-Dec-20X2)
Floating rate option	EUR-Euribor fixed on the second local business day preceding the last floating amount payment date. The floating rate for the initial period would be determined two local business days prior to the effective date
Designated maturity	3 months
Spread	Plus 120 bps (1.2%)
Floating rate day count fraction	Actual/360
Dividend amount part	
Dividend amount	100% of the paid amount
Dividend payer	Party B (Gigabank)
Dividend receiver	Party A (ABC)
Dividend period	The period commencing on, and including, the third scheduled trading day preceding the effective date and ending on, and including, the termination date
Dividend payment date	The date of the payment of the dividend by the issuer of the shares
Reinvestment of dividends	Not applicable

The **trade date** is the date on which the two counterparties to the equity swap agree on the terms of the equity swap. In our case, ABC and Gigabank agreed on the equity swap terms on 12 December 20X1.

The **effective date** is the date on which the equity swap starts. In our case, the equity swap started on 15 December 20X1. On this date, Gigabank had already put in place its initial hedge. Also, the interest period usually starts on the effective date.

The **termination date** is the date on which the equity swap ceases to exist. In our case, the termination date was 15 December 20X2. In other words, the equity swap term was one year.

The equity amount part

The **equity amount part** represents the information regarding the equity underlying and the way the equity swap will be settled. In our example, the equity swap will be **physically settled**, meaning that on the **settlement date** the equity amount receiver would be receiving the **number of shares** of DTE in exchange for the **equity notional amount**. The equity notional amount was determined as follows:

$$\text{Equity notional amount} = \text{Number of shares} \times \text{Initial price}$$
$$\text{Equity notional amount} = 10 \text{ million shares} \times 14$$
$$= \text{EUR } 140 \text{ million}$$

The floating amount part

The **notional amount** as defined in the floating amount part of the equity swap confirmation is the underlying quantity upon which floating amount payment obligations are computed. Normally this notional amount equates to the equity notional amount. Thus, in our case the notional amount was EUR 140 million.

In our example, ABC will pay Euribor 3-month plus 120 basis points at the end of each quarter. The Euribor 3-month rate is set two business days prior to the beginning of the interest period. The interest (i.e., the **floating amount**) to be paid at the end of each quarterly period is calculated as:

$$\text{Floating amount} = \text{Notional amount} \times (\text{Euribor 3-month} + 1.20\%) \times \text{Actual days}/360$$

The "360" that appears in the formula is the denominator of the **floating rate day count**.

Actual days are the number of calendar days in the interest period.

Let us assume that the Euribor 3-month two business days prior to 15 December 20X1 was fixed at 3.30%. Let us assume further that there were 91 calendar days in the period from 15 December 20X1 to 15 March 20X2. At the end of the interest period (i.e., on 15 March 20X2) ABC paid the following floating amount (see Figure 1.8):

$$\text{Floating amount} = 140 \text{ million} \times (3.30\% + 1.20\%) \times 91/360 = \text{EUR } 1{,}592{,}500$$

The dividend amount part

The **dividend amount** reflects the benefits of carrying the underlying stock, mainly cash dividends distributed to the underlying shares. If an ex-dividend date falls in the dividend

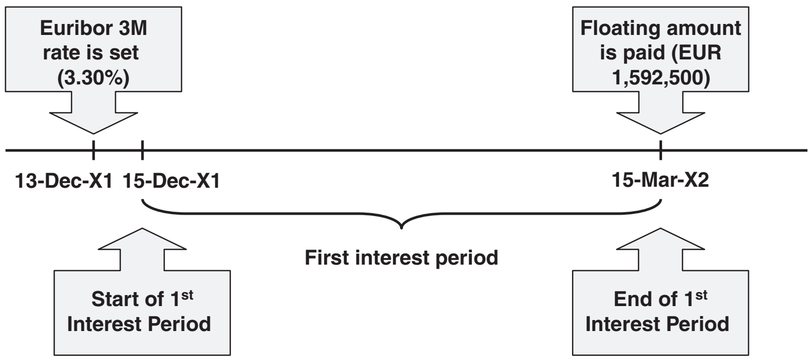

Figure 1.8 First floating amount period.

period (i.e., in our case the period between the effective date and the termination date), an amount equal to a percentage of the paid amount by the issuer of the stock is paid by the dividend payer – Gigabank – to the dividend receiver – ABC. In our example, Gigabank has to pay 100% of the ordinary and extraordinary cash or stock dividends declared in the currency of the company announcement. Stock/scrip dividends are included at the equivalent cash amount. The dividend amount is paid on the same date on which the issuer of the shares pays the dividend to the shareholders. Note that in this example, dividends are not reinvested.

In our case, the "dividend amount" was defined as "100% of the paid amount". When an issuer declares a dividend, the amount declared is called the "**gross dividend**" (i.e., the paid amount). The issuer of the shares – DTE – may levy a withholding tax when distributing dividends to Gigabank. As an example, let us assume that Gigabank receives what is called a "**net dividend**", for example 85% of the gross dividend. As a result, Gigabank would incur an additional cost if it pays 100% of the gross dividends to ABC. In order to not incur this additional cost, the dividend payer – Gigabank – pays to the dividend receiver – ABC – a dividend amount net of any withholding taxes or other taxes or duties in connection with the receipt of such dividend, equating to the dividend it receives from DTE. In this example, the "dividend amount" would be defined as "85% of the paid amount".

Reinvestment of Dividends

Sometimes, in a total return equity swap dividends are reinvested until maturity or until the next equity notional is reset. The reinvestment does not assume that the dividend proceeds are invested in money market instruments. Instead, the dividend proceeds are assumed to be reinvested in the same stock, at the closing of the ex-dividend date. Let us assume that the initial price of an equity swap on 10 million shares of a specific stock was USD 17.90. The equity swap effective date was 15 December 20X1 and its termination date was 15 December 20X2. During the life of the equity swap, the underlying stock paid two cash dividends:

- A gross dividend of USD 0.5 per share. Its ex-dividend date was 10 March 20X2. The closing price of the stock on 10 March 20X2 was USD 18.10.
- A gross dividend of USD 0.6 per share. Its ex-dividend date was 10 September 20X2. The closing price of the stock on 10 September 20X2 was USD 18.30.

The stock closing price on the equity swap valuation date was USD 18.50. The settlement price needs to be multiplied by the adjustment factors related to each dividend:

$$\text{Settlement price} = \text{USD } 18.50 \times \text{Adjustment factor}_1 \times \text{Adjustment factor}_2$$
$$\text{Adjustment factor} = 1 + [\text{Dividend}/(\text{Share price on ex-dividend date})]$$
$$\text{Adjustment factor}_1 = 1 + (0.50/18.10) = 1.02762$$
$$\text{Adjustment factor}_2 = 1 + (0.60/18.30) = 1.03279$$
$$\text{Settlement price} = \text{USD } 18.50 \times 1.02762 \times 1.03279 = \text{USD } 19.63$$

The cash settlement amount would then be USD 17.3 million [= 10 million shares × (19.63 − 17.90)] to be paid by the equity amount payer to the equity amount receiver.

1.2.4 Case Study: Cash-settled Total Return Equity Swap on Deutsche Telekom

In this subsection, I will cover in detail the mechanics of a cash-settled total return equity swap on a stock using a similar example. In this case, ABC believes that the stock of Deutsche Telekom (DTE) will increase in value over the next 12 months. ABC does not own DTE stock at inception and it is not interested in acquiring any DTE shares at maturity. ABC acquires the economic equivalent of a long position in the stock with a cash-settled total return equity swap.

Main Terms of the Equity Swap

The main terms of the equity swap are shown in the following table:

Cash-settled Total Return Equity Swap Terms	
Party A	ABC Corp.
Party B	Gigabank
Trade date	12-December-20X1
Effective date	15-December-20X1
Termination date	15-December-20X2
Underlying currency	EUR
Equity amount part	
Equity amount payer	Party B (Gigabank)
Equity amount receiver	Party A (ABC Corp.)
Shares	Deutsche Telekom common shares
Exchange	Euronext
Calculation agent	Party B (Gigabank)
Cash settlement	Applicable
Number of shares	10 million
Initial price	EUR 14.00
Equity notional amount	EUR 140 million Number of shares × Initial price
Settlement price	The price per share at the valuation time on the valuation date
Valuation time	At the close of trading on the exchange
Valuation date	12-December-20X2 (the third exchange business day preceding the settlement date)
Settlement date	15-December-20X2 (the termination date)

Cash-settled Total Return Equity Swap Terms

Floating amount part	
Floating amount payer	Party A (ABC Corp.)
Notional amount	The equity notional amount
Payment dates	At the end of each quarter (15-Mar-20X2, 15-Jun-20X2, 15-Sep-20X2 and 15-Dec-20X2)
Floating rate option	EUR-Euribor fixed on the second local business day preceding the last floating amount payment date. The floating rate for the initial period would be determined two local business days prior to the effective date
Designated maturity	3 months
Spread	Plus 120 bps (1.2%)
Floating rate day count fraction	Actual/360
Dividend amount	
Dividend amount	100% of the paid amount
Dividend payer	Gigabank
Dividend receiver	ABC Corp.
Dividend period	The period commencing on, and including, the third scheduled trading day preceding the effective date and ending on, and including, the valuation date
Dividend payment date	The date of the payment of the dividend by the issuer of the shares
Reinvestment of dividends	Not applicable

Flows of the Transaction

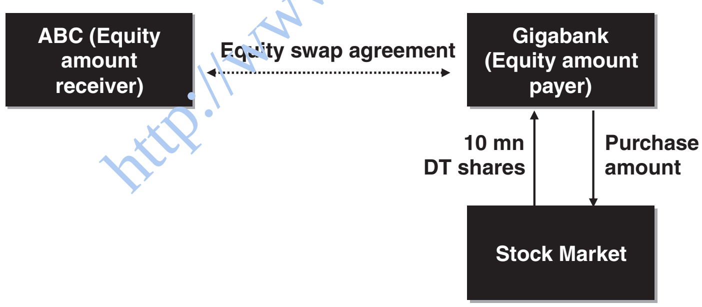

Figure 1.9 Flows on trade date.

The flows of the transaction on the trade date are as follows (see Figure 1.9):

- ABC and Gigabank enter into the cash-settled total return equity swap on 10 million shares of DTE. The initial price is set at EUR 14.00.
- Gigabank initially hedges its position by buying 10 million DTE shares in the stock market.

The flows of the transaction during the life of the equity swap are as follows (see Figure 1.10):

- ABC pays an interest, the floating amount, to Gigabank based on the EUR 140 million equity swap notional. The floating amount is identical to the physically settled equity swap covered in our previous case.

Figure 1.10 Flows during the life of the equity swap.

- Gigabank pays to ABC an amount, the dividend amount, equivalent to the dividends distributed to the underlying DTE shares. This amount is paid on the same date the dividends are distributed by DTE to its shareholders. The dividend amount is identical to the physically settled equity swap covered in our previous case.

The flows of the transaction at maturity (i.e., on settlement date of the equity swap) are as follows (see Figure 1.11):

- Gigabank unwinds its hedge, selling 10 million shares of DTE in the market.
- ABC and Gigabank settle the equity swap after calculating the settlement amount.

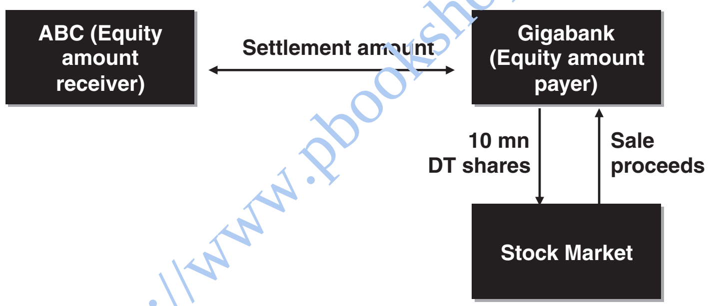

Figure 1.11 Flows on the settlement date of the equity swap.

On the valuation date, 12 December 20X2, the settlement amount would be calculated as:

Settlement amount = Number of shares × (Settlement price – Initial price)
Settlement amount = EUR 10 million × (Settlement price – 14.00)

The settlement price would be the closing price of DTE stock on the valuation date – 13 December 20X2.

- If the settlement amount is positive, the equity amount payer – Gigabank – would pay to the equity amount receiver – ABC – the settlement amount. For example, if the settlement price is EUR 17.00, ABC would receive from Gigabank EUR 30 million [= 10 million × (17.00 – 14.00)].
- If the settlement amount is negative, the equity amount receiver – ABC – would pay to the equity amount payer – Gigabank – the absolute value of the settlement amount. For example, if the settlement price is EUR 12.00, Gigabank would receive from ABC EUR 20 million [= Absolute value of 10 million × (12.00 – 14.00)].

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1.2.5 Determination of the Initial Price

Commonly the bank facilitating the equity swap, Gigabank in our case, would initially be hedging its market exposure under the equity swap. The start of the execution of the hedge would start on trade date. Due to the fact that the delta of an equity swap is either 100% or minus 100%, the bank needs initially either to buy or to sell the **number of shares**. In our case, because Gigabank was exposed to a rising stock price and the number of shares was 10 million, Gigabank needed to buy 10 million DTE shares at the beginning of the transaction.

The **initial price** is commonly set according to one of the following three ways:

1. The acquisition/sale price of the shares executed in one block.

   In our previous case, the physically settled total return equity swap on Deutsche Telekom, Gigabank acquired the shares to initially hedge its position directly from ABC in one single transaction. In that case, the shares were acquired by Gigabank at EUR 14 per share. Therefore, the initial price was set at EUR 14 because it was the price at which Gigabank put in place its initial hedge. In this situation, the initial price is known on the trade date. No initial hedging period is needed.
2. The volume-weighted average price per share at which the bank puts in place its initial hedge.

   In most transactions, the initial hedge is put in place in the market during a period called the initial hedging period. When the size of the transaction is large relative to the average daily volume of the underlying, the initial hedge is executed during several days (see Figure 1.12). Let us assume that Gigabank had to buy the shares in the market. In order to not affect the stock price, and to comply with stock exchange regulations, Gigabank would try to not exceed 20% of the daily volume for DTE stock. If the daily volume average of DTE stock is 20 million shares, Gigabank would need 2.5 days to buy the 10 million DTE shares [= 10 million/(20 million × 20%)]. More formally, the initial price would be defined as: “the weighted average execution price at which Gigabank buys the shares between the trade date and the last day during which Gigabank puts in place its hedge position, on a best effort basis as determined and notified by the calculation agent. This last day will be the effective date. Gigabank shall notify ABC of the weighted average execution price at which Gigabank buys the shares on a daily basis whilst it is putting in place its hedge position.”

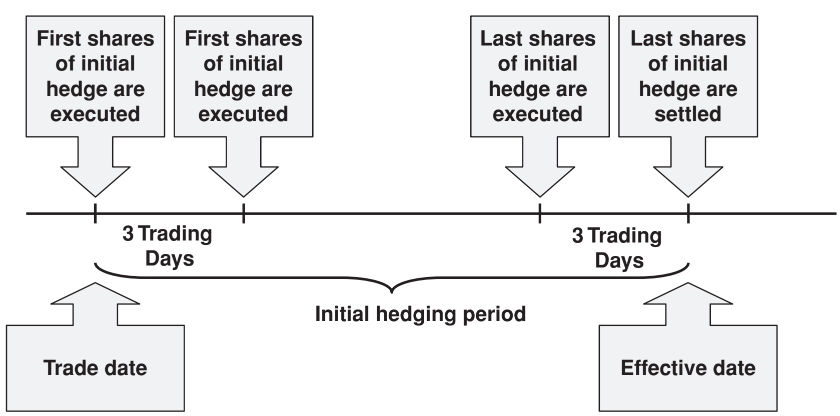

Figure 1.12 Initial hedging period.

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The problem with this method of setting the initial price is that ABC is exposed to a poor execution by Gigabank. If Gigabank is not careful, the execution price might underperform the volume-weighted average price of the DTE stock during the period.

The effective date becomes the date on which the last shares of the initial hedge are settled. Because Gigabank has acquired shares during the initial hedging period, it has to finance the acquisition of these shares. As a result, ABC has to compensate Gigabank for the expense incurred by paying to Gigabank an initial floating amount on the effective date. This initial floating amount is calculated as the sum, for each day of the initial hedging period, of the acquisition expense:

$$\text{Acquisition expense} = (\text{Shares acquired}) \times (\text{Initial floating rate} + \text{Spread}) \times \text{Days}/360$$

where:

Shares acquired = Number of shares settled on such day
Initial floating rate = Euribor rate fixed two days prior to the first day of the initial hedging period
Days = Number of calendar days from such day until the effective date

3. The average of the VWAP over a number of days.

Less frequent is to set the reference price as the average of an official price of the stock during a predetermined number of days. The most widely used official price is the volume-weighted average price (VWAP) of the day. For example, in our case the initial price could have been defined as "the arithmetic average of the daily volume-weighted average price over three exchange business days after the trade date". The advantage for ABC is that the calculation of the initial price is transparent, not depending on how Gigabank executes the hedge.

In our case, Gigabank puts in place its initial hedge by buying 10 million DTE shares in the market. Let us call the average price at which Gigabank buys these 10 million shares the "hedging price". Therefore, Gigabank is running the risk of any deviations of the hedging price relative to the average VWAP during the pre-agreed three days. It obliges Gigabank to acquire the shares in each of the three days, trying to mimic the VWAP for that day. If this risk is substantial, Gigabank would take it into account by requesting a higher spread from ABC.

1.2.6 Determination of the Settlement Price

One moment before the valuation time on the valuation date, the bank counterparty to the equity swap is either long or short the underlying number of shares. In our case, Gigabank was long 10 million shares of DTE. Therefore, at the valuation time on the valuation date Gigabank would need to sell 10 million shares immediately.

When the settlement mode is physical settlement, on the settlement date Gigabank will sell the 10 million shares to ABC at the initial price. In this case, the hedge unwind by Gigabank is not an issue, as it is executed in one block.

When the settlement mode is cash settlement, on the settlement date Gigabank and ABC will settle a cash amount (the settlement amount) but no shares will be exchanged between the two counterparties. At the same time, Gigabank will be unwinding its hedge by selling

10 million shares in the market. As a result, Gigabank would be exposed to any deviation of the price at which it sells the stock relative to the settlement price. Commonly, in a cash-settlement equity swap the settlement price is defined in one of the following ways:

- The closing price of the shares on a specific day. The valuation time is defined as "the close of the regular trading session on the valuation date". In order to mitigate its risk, Gigabank would need to sell the 10 million DTE shares at the closing, which can have a substantial impact on the stock closing price. Therefore, this definition of the settlement price makes sense when the number of shares of the underlying is small relative to its average daily volume.
- The arithmetic average of the daily VWAP over a pre-established number of days. In our case, it is expected that the hedge unwind would take three trading days. The settlement price would then be defined as "the arithmetic average of the daily volume-weighted average price over three exchange business days prior to and including the valuation date". As discussed in the previous subsection on determination of the initial price, in order to mitigate its risk, Gigabank would need to sell the shares during each of the pre-established days, trying to replicate the VWAP of such day. As each day Gigabank is running the risk of not achieving the VWAP, Gigabank would take it into account by requesting a higher spread from ABC.
- The volume-weighted average price per share at which the bank unwinds its hedge. More formally, "the weighted average execution price at which Gigabank sells the shares of its hedge position during the three exchange business days prior to, and including, the valuation date, on a best effort basis as determined and notified by the calculation agent. Gigabank shall notify ABC of the weighted average execution price at which Gigabank sells the shares on a daily basis whilst it is unwinding its hedge position." With this definition of the settlement price, Gigabank is not exposed to any market risks related to the unwinding of its hedge. However, ABC is exposed to a lower than expected sale price due to a bad execution by Gigabank. A lower than expected sale price will imply a lower than expected settlement price, diminishing ABC's profit (or enlarging its loss).

1.2.7 Equity Notional Resets

In our previous case, the equity notional was settled at the end of the equity swap life. It is not unusual that the equity notional is reset periodically, commonly coinciding with the payment of the floating amount. Long-term equity swaps generally reset quarterly, with the two counterparties exchanging payments based on the previous three months' returns. This makes the long-term equity swap equivalent to a series of three-month equity swaps. Of course, the inclusion of equity notional resets makes sense only for cash-settlement equity swaps. Equity swaps with equity notional resets help to reduce the counterparty credit risk associated with long-term transactions.

1.2.8 Case Study: Total Return Equity Swap on EuroStoxx 50

Although quite similar to a cash-settled equity swap on a stock, an equity swap on a stock index has some specific features. In this case I will try to address these peculiarities. Let us assume that ABC Corp. has the view that the main European stock index, the EuroStoxx 50 is going to have a strong performance during the next 12 months. In the following case, ABC obtains through a cash-settled total return equity swap an economic exposure to the EuroStoxx 50 index without physically owning the stocks members of the index. The main terms of the equity swap are shown in the following table:

Terms of a Total Return Equity Swap on a Stock Index	
Party A	ABC Corp.
Party B	Gigabank
Trade date	12-December-20X1
Effective date	15-December-20X1
Termination date	15-December-20X2
Underlying currency	EUR
Equity amount part	
Underlying index	EuroStoxx 50 index
Type of return	Total return
Calculation agent	Party B (Gigabank)
Number of baskets	100,000 (for avoidance of doubt, the number of baskets is in EUR being 10,000 contracts × 10 EUR tick value)
Initial index level ($Index_0$)	3,500
Equity notional amount	EUR 350 million (Number of baskets × $Index_0$)
Equity amount	Equity notional amount × $[(Index_T - Index_0)/Index_0]$
Equity amount payer	Party B (Gigabank)
Equity amount receiver	Party A (ABC Corp.)
$Index_T$	Level of the index at the closing of the valuation date
Cash settlement	Applicable
Settlement currency	EUR
Valuation date	12-December-20X2
Settlement date	15-December-20X2
Floating amount part	
Floating amounts payer	Party A (ABC Corp.)
Floating amount	The equity notional amount
Payment dates	At the end of each quarter (15-Mar-20X2, 15-Jun-20X2, 15-Sep-20X2 and 15-Dec-20X2)
Floating rate option	EUR-Euribor fixed on the second local business day preceding the last floating amount payment date. The floating rate for the initial period would be determined two local business days prior to the effective date
Designated maturity	3 months
Spread	Plus 100 bps (1.0%)
Floating rate day count fraction	Actual/360
Dividend amount part	
Dividend amount payer	Party B (Gigabank)
Dividend amount receiver	Party A (ABC Corp.)
Dividend amounts	Number of baskets × Dividend per basket
Dividend period	Each quarterly period, coinciding with each interest period

Flows of the Transaction

The flows of the transaction on the trade date are as follows (see Figure 1.13):

- ABC and Gigabank enter into the cash-settled total return equity swap on 100,000 baskets of the EuroStoxx 50 index. The initial index level is set at EUR 3,500.

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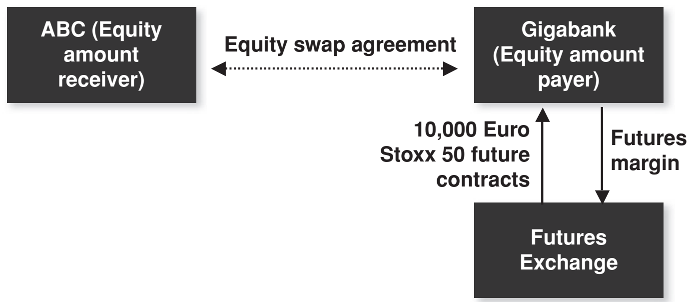

Figure 1.13 Flows on trade date.

- Gigabank initially hedges its position by buying 10,000 contracts of EuroStoxx 50 index futures. Gigabank posts initial margin at the futures exchange. This flow is not part of the equity swap.

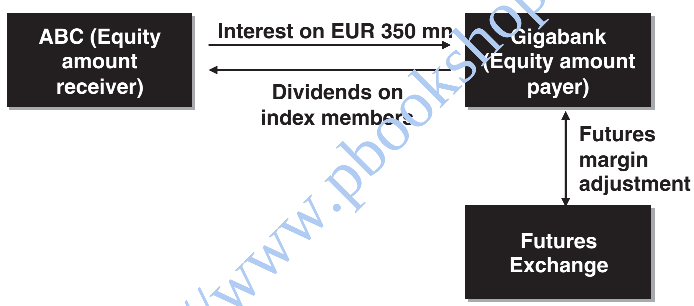

Figure 1.14 Flows during the life of the equity swap.

The flows of the transaction during the life of the equity swap are as follows (see Figure 1.14):

- ABC pays an interest, the floating amount, to Gigabank. ABC pays Euribor 3-month + 100 bps on the EUR 350 million equity notional amount.
- Gigabank pays to ABC an amount, the dividend amount, equivalent to the dividends distributed to the underlying stocks of the EuroStoxx 50 index during the dividend period. This amount is paid on the same date the floating amount is paid. The dividend amount is calculated by multiplying the "number of baskets" by the "dividend per basket" (to be defined later).
- Gigabank, on a daily basis, posts additional margin (or recovers some of the margin posted) to the futures exchange. This flow is not part of the equity swap.

The flows of the transaction at maturity (i.e., on the settlement date of the equity swap) are as follows (see Figure 1.15):

- Gigabank unwinds its hedge, selling 10,000 contracts of EuroStoxx 50 index futures. Gigabank receives the margin posted at the futures exchange.
- ABC and Gigabank settle the equity swap after calculating the equity amount. If the equity amount is positive, the index has appreciated and Gigabank pays to ABC the equity amount.

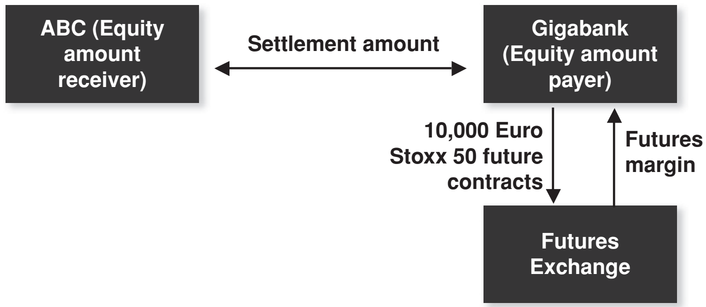

Figure 1.15 Flows on the settlement date of the equity swap.

If, on the other hand, the equity amount is negative, the index has depreciated and ABC pays to Gigabank the absolute value of the equity amount.

On valuation date, 12 December 20X2, the equity amount would be calculated as:

$$\text{Equity amount} = \text{Equity notional amount} \times [(\text{Index}_T - \text{Index}_0)/\text{Index}_0]$$

or also:

$$\text{Equity amount} = \text{Number of baskets} \times (\text{Index}_T - \text{Index}_0)$$
$$\text{Equity amount} = \text{EUR } 100{,}000 \times (\text{Index}_T - 3{,}500)$$

Index_T is the closing price of the EuroStoxx 50 index on the valuation date – 13 December 20X2.

- If the equity amount was positive, the equity amount payer – Gigabank – would pay to the equity amount receiver – ABC – the equity amount on the settlement date. For example, if Index_T was 4,000, ABC would receive from Gigabank EUR 50 million [= 100,000 × (4,000 – 3,500)].
- If the equity amount was negative, the equity amount receiver – ABC – would pay to the equity amount payer – Gigabank – the absolute value of the equity amount on the settlement date. For example, if Index_T is 3,000, Gigabank would receive from ABC EUR 50 million [= Absolute value of 100,000 × (3,000 – 3,500)].

Calculation of the Dividend per Basket

As seen earlier, Gigabank pays to ABC an amount, the dividend amount, equivalent to the dividends distributed to the underlying stocks of the EuroStoxx 50 index during the dividend period. In our case, the dividend period was each quarterly period during the life of the equity swap. Thus, the dividend amount is paid on the same date the floating amount is paid (i.e., at the end of each quarterly period). The dividend amount is calculated as:

$$\text{Dividend amount} = \text{Number of baskets} \times \text{Dividend per basket}$$

The "Dividend per basket" is calculated as:

$$\sum_s \sum_i \frac{n_{is} \times d_{is}}{D_i}$$

where:

i means each weekday (each a "Relevant day$_i$") in the relevant dividend period.
s means, in respect of each Relevant day$_i$, each share (each a "Share") that is comprised in the index on such a relevant day.
d_{is} means, in respect of each Share$_i$ and a Relevant day$_t$:
(a) if an ex-dividend date in respect of such Share$_i$ falls on such Relevant day$_t$, an amount equal to the relevant dividend in respect of such Share$_i$ and such Relevant day$_t$; or
(b) otherwise, zero (0).
n_{is} means, in respect of each Share$_i$ and a Relevant day$_t$, the number of such free-floating Share$_i$ comprised in the index, as calculated and published by the index sponsor on such Relevant day$_t$.
D_i means, in respect of each Relevant day$_t$, the official index divisor, as calculated and published by the index sponsor on such Relevant day$_t$.
"Relevant dividends" means "All dividends" for every period, for member stocks of the index that trade ex-dividend in that period, and for which the index sponsor does not adjust the index. If the index sponsor adjusts the index for part of a dividend, the remaining part is considered the relevant dividend.
"All dividends" means:

- "Declared cash dividend" – an amount per Share$_i$ as declared by the issuer of such Share$_i$ where the ex-dividend date falls on such Relevant day$_t$, before the withholding or deduction of taxes at the source by or on behalf of any applicable authority having power to tax in respect of such a dividend, and shall exclude any imputation or other credits, refunds or deductions granted by any applicable authority having power to tax in respect of such dividend and any taxes, credits, refunds or benefits imposed, withheld, assessed or levied thereon and/or
- "Declared cash equivalent dividend" – the cash value of any stock dividend declared by the issuer of such Share$_i$ where the ex-dividend date falls on such Relevant day$_t$ (or, if no cash value is declared by the relevant issuer, the cash value of such stock dividend as determined by the calculation agent), calculated by reference to the opening price of such ordinary shares on the ex-dividend date applicable to that stock dividend.

1.2.9 Compo Equity Swaps

Through a compo equity swap, an investor can express a view that foreign stocks will rise or decline, taking also a position in the related foreign currency. A compo swap equity swap is equivalent to a direct foreign stock investment executed in an equity swap form. Therefore, a compo equity swap is an equity swap in which the underlying is denominated in a currency (the foreign currency) other than the currency in which the equity swap is denominated (the domestic currency). The initial and final values of the underlying are denominated in the foreign currency and are converted into the domestic currency using the exchange rate prevailing at the time the initial and final values are calculated. Although the equity swap is denominated in the investor's base currency, the investor is exposed to currency risk.

Let us assume that ABC, a EUR-based entity, thinks that the S&P 500 index will rise strongly during the next 12 months. The S&P 500 index is denominated in USD. ABC also thinks that the USD will appreciate relative to the EUR. ABC enters into a compo price return equity swap with Gigabank with the following terms:

Terms of a Compo Price Return Equity Swap on a Stock Index	
Party A	ABC Corp.
Party B	Gigabank
Trade date	12-December-20X1
Effective date	15-December-20X1
Termination date	15-December-20X2
Underlying currency	USD
Initial exchange rate (FX_0)	1.2500
Final exchange rate (FX_T)	The USD/EUR exchange rate as published on Bloomberg WMCO page on the valuation date at 04:00 p.m. London time
Equity amount part	
Underlying index	S&P 500 index
Type of return	Price return
Calculation agent	Party B (Gigabank)
Number of baskets	100,000
Initial index level ($Index_0$)	USD 1,200
Equity notional amount	EUR 96 million (Number of baskets × $Index_0/FX_0$)
Equity amount	Number of baskets × ($Index_T/FX_T$ – $Index_0/FX_0$)
Equity amount payer	Party B (Gigabank)
Equity amount receiver	Party A (ABC Corp.)
$Index_T$	Level of the index at the closing of the valuation date
Cash settlement	Applicable
Settlement currency	EUR
Valuation date	12-December-20X2
Settlement date	15-December-20X2
Floating amount part	
Floating amounts payer	Party A (ABC Corp.)
Floating amount	The equity notional amount
Payment dates	At the end of each quarter (15-Mar-20X2, 15-Jun-20X2, 15-Sep-20X2 and 15-Dec-20X2)
Floating rate option	EUR-Euribor fixed on the second local business day preceding the last floating amount payment date. The floating rate for the initial period would be determined two local business days prior to the effective date
Designated maturity	3 months
Spread	Plus 20 bps (0.20%)
Floating rate day count fraction	Actual/360
Dividend amount part	**Not applicable**

The mechanics of the compo equity swap are quite similar to the cash-settled equity swap case covered previously.

The flows of the transaction on the trade date are as follows:

- ABC and Gigabank enter into the cash-settled compo price return equity swap on 100,000 baskets of the S&P 500 index. The initial index level ($Index_0$) is set at USD 1,200. The initial USD/EUR exchange rate (FX_0) is set at 1.2500.
- Gigabank initially hedges its position by buying 400 contracts of S&P 500 index futures. Gigabank posts initial margin at the futures exchange.

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The flows of the transaction during the life of the equity swap are as follows:

- ABC pays an interest, the floating amount, to Gigabank. ABC pays Euribor 3-month + 20 bps on the EUR 96 million equity notional amount. Note that although the underlying stock index is denominated in USD, the floating amount is denominated in EUR.
- In this example there is no dividend amount, as it is a price return swap. Therefore, Gigabank keeps the dividends distributed to the underlying stocks of the S&P 500 index during the life of the equity swap. As a result, Gigabank compensates ABC by charging a lower spread (20 basis points in our case).

The flows of the transaction at maturity (i.e., on the settlement date of the equity swap) are as follows:

- Gigabank unwinds its hedge, selling 400 contracts of the S&P 500 index futures. Gigabank receives back the margin posted at the futures exchange.
- ABC and Gigabank settle the equity swap after calculating the equity amount. If the equity amount is positive, Gigabank pays to ABC the equity amount. If, on the other hand, the equity amount is negative, ABC pays to Gigabank the absolute value of the equity amount.

On the valuation date, 12 December 20X2, the equity amount would be calculated as:

$$\text{Equity amount} = \text{Number of baskets} \times (\text{Index}_T/\text{FX}_T - \text{Index}_0/\text{FX}_0)$$

Index_T is the closing price of the S&P 500 index on the valuation date – 12 December 20X2. FX_T is the USD/EUR exchange rate on the valuation date.

If the equity amount was positive, the equity amount payer – Gigabank – would pay to the equity amount receiver – ABC – the equity amount on the settlement date. For example, if Index_T is 1,600 and FX_T is 1.10, ABC would receive from Gigabank EUR 49,454,545.46 [= 100,000 × (1,600/1.10 – 1,200/1.25)].

If the equity amount was negative, the equity amount receiver – ABC – would pay to the equity amount payer – Gigabank – the absolute value of the equity amount on the settlement date. For example, if Index_T is 1,000 and FX_T is 1.30, Gigabank would receive from ABC EUR 19,076,923.08 (= Absolute value of [100,000 × (1,000/1.30 – 1,200/1.25)]).

It is important to note that ABC is exposed to both the S&P 500 index and the USD/EUR exchange rate performances. ABC benefits from a rising S&P 500 index and/or from a declining USD/EUR FX rate. Conversely, ABC loses from a declining S&P 500 index and/or from a rising USD/EUR FX rate.

1.2.10 Quanto Equity Swaps

Through a quanto equity swap, an investor can express a view that foreign stocks will rise or decline, without taking a position in the foreign currency. Contrast this with a compo equity swap, in which the investor takes long or short exposure to the foreign stocks and the foreign currency. Therefore, a quanto equity swap is an equity swap in which the underlying is denominated in a currency (the foreign currency) other than that in which the equity swap is denominated (the domestic currency). The final value of the underlying is denominated in the foreign currency and is converted into the domestic currency using the exchange rate prevailing at inception. As a result, the investor is not exposed to currency risk.

Let us assume that ABC, a EUR-based entity, thinks that the S&P 500 index will rise strongly during the next 12 months. ABC is unsure of the USD behavior relative to the EUR, preferring to take currency exposure out of its position. ABC enters into a quanto price return equity swap with Gigabank with the following terms:

Terms of a Quanto Price Return Equity Swap on a Stock Index	
Party A	ABC Corp.
Party B	Gigabank
Trade date	12-December-20X1
Effective date	15-December-20X1
Termination date	15-December-20X2
Underlying currency	USD
Initial exchange rate (FX_0)	1.2500
Equity amount part	
Underlying index	S&P 500 index
Type of return	Price return
Calculation agent	Party B (Gigabank)
Number of baskets	100,000
Initial index level ($Index_0$)	USD 1,200
Equity notional amount	EUR 96 million (Number of baskets × $Index_0/FX_0$)
Equity amount	Number of baskets × ($Index_T/FX_0$ – $Index_0/FX_0$)
Equity amount payer	Party B (Gigabank)
Equity amount receiver	Party A (ABC Corp.)
$Index_T$	Level of the index at the closing of the valuation date
Cash settlement	Applicable
Settlement currency	EUR
Valuation date	12-December-20X2
Settlement date	15-December-20X2
Floating amount part	
Floating amounts payer	Party A (ABC Corp.)
Floating amount	The equity notional amount
Payment dates	At the end of each quarter (15-Mar-20X2, 15-Jun-20X2, 15-Sep-20X2 and 15-Dec-20X2)
Floating rate option	EUR-Euribor fixed on the second local business day preceding the last floating amount payment date. The floating rate for the initial period would be determined two local business days prior to the effective date
Designated maturity	3 months
Spread	Plus 20 bps (0.20%)
Floating rate day count fraction	Actual/360
Dividend amount part	**Not applicable**

The flows of the transaction on the trade date are as follows:

- ABC and Gigabank enter into the cash-settled quanto price return equity swap on 100,000 baskets of the S&P 500 index. The initial index level ($Index_0$) is set at USD 1,200. The initial USD/EUR exchange rate (FX_0) is set at 1.2500.
- Gigabank initially hedges its position by buying 400 contracts of S&P 500 index futures. Gigabank posts initial margin at the futures exchange.

The flows of the transaction during the life of the equity swap are as follows:

- ABC pays an interest, the floating amount, to Gigabank. ABC pays Euribor 3-month + 20 bps on the EUR 96 million equity notional amount. Note that although the underlying stock index is denominated in USD, the floating amount is denominated in EUR.
- In this example there is no dividend amount, as it is a price return swap. Therefore, Gigabank keeps the dividends distributed to the underlying stocks of the S&P 500 index during the life of the equity swap. As a result, Gigabank compensates ABC by charging a lower spread (20 basis points in our case).

The flows of the transaction at maturity (i.e., on the settlement date of the equity swap) are as follows:

- Gigabank unwinds its hedge, selling 400 contracts of the S&P 500 index futures. Gigabank receives back the margin posted at the futures exchange.
- ABC and Gigabank settle the equity swap after calculating the equity amount. If the equity amount is positive, the index has appreciated and Gigabank pays to ABC the equity amount. If, on the other hand, the equity amount is negative, the index has depreciated and ABC pays to Gigabank the absolute value of the equity amount.

On the valuation date, the equity amount would be calculated as:

$$\text{Equity amount} = \text{Number of baskets} \times (\text{Index}_T/\text{FX}_0 - \text{Index}_0/\text{FX}_0)$$

Index_T is the closing price of the S&P 500 index on the valuation date – 12 December 20X2. FX_0 was the USD/EUR exchange rate – 1.2500 – on the effective date – 15 December 20X1.

- If the equity amount was positive, the equity amount payer – Gigabank – would pay to the equity amount receiver – ABC – the equity amount on the settlement date. For example, if Index_T is 1,600, ABC would receive from Gigabank EUR 32 million [= 100,000 × (1,600/1.25 – 1,200/1.25)].
- If the equity amount was negative, the equity amount receiver – ABC – would pay to the equity amount payer – Gigabank – the absolute value of the equity amount on the settlement date. For example, if Index_T is 1,000, Gigabank would receive from ABC EUR 16 million (= Absolute value of [100,000 × (1,000/1.25 – 1,200/1.25)]).

It is important to note that ABC is exposed only to the S&P 500 index and not to the USD/EUR exchange rate. ABC benefits from a rising S&P 500 index, independently of the behavior of the USD/EUR FX rate. Conversely, ABC loses from a declining S&P 500 index, independently of the behavior of the USD/EUR FX rate. Thus, a quanto equity swap is a valuable tool for equity investors to manage their exposure when investing in foreign equity markets without being saddled with foreign currency exposure.

1.2.11 Uses of Equity Swaps

Equity swaps can be particularly useful in a number of situations, for example:

- **To diversify a portfolio**. Equity swaps give economic exposure to a stock, basket of stocks or an equity index without legal ownership of the underlying. For example, an asset manager is bullish on a specific sector and wants to get a long exposure to the sector without buying the underlying stocks. The asset manager can buy an equity swap on the index that best tracks that sector. Note, however, that certain benefits of physical ownership are not obtained, such as the voting rights. Unlike a futures position, an equity swap does not require rolling positions.

- **To protect a portfolio**. Equity swaps reduce economic exposure to a stock, basket of stocks or an equity index while maintaining legal ownership of the underlying. For example, an asset manager wants to underweight a specific sector but does not want to sell the underlying stocks. The asset manager can go short an equity swap on the index that best tracks that sector.
- **To finance a portfolio**. An investor may seek to raise financing without using the traditional debt capital markets. The investor sells its portfolio and maintains an economic exposure to the portfolio by going long an equity swap on it. By selling the portfolio, the investor raises cash.
- **To obtain synthetic borrow and lending**. Where investors are unable to borrow or lend stocks, they can use equity swaps to borrow or lend stock synthetically. For example, say a certain stock is difficult to borrow, and an investor is seeking to benefit from lending the stock out. If an investor is unable to lend stocks, he/she can sell the stock and enter into a long total return swap. The swap counterparty can lend the stock out, being able to pass on the benefit by charging a lower financing rate to the investor.

1.3 STOCK LENDING AND BORROWING

Sometimes the benefit from lending stock out can be significant. For example, a stock may be expensive to borrow because it is involved in a merger or acquisition. If an investor in the stock does not lend the stock out, he/she will forgo the opportunity to outperform without taking any major additional risk.

1.3.1 Stock Lending and Borrowing

A stock lending and borrowing, or securities lending and borrowing, agreement is a contract between two counterparties whereby one counterparty – the stock borrower – borrows a stock from the other counterparty – the stock lender – with a commitment by the borrower to return equivalent stock at a future date. In return for borrowing the stock, the borrower pays a fee – the lending fee – and posts collateral.

A securities lending and borrowing agreement is formalized through a confirmation. The confirmation is generally legally subject to the terms and clauses of the Global Master Securities Lending Agreement (GMSLA) signed between the two counterparties that sets out their obligations.

In securities lending, the lender effectively retains all the benefits of ownership, other than the voting rights. The borrower can use the securities as required – perhaps by selling them or by lending them on to another party – but is liable to the lender for all the benefits distributed to the underlying shares, such as dividends or stock bonuses. The voting rights are transferred to the borrower in a stock lending transaction. If the lender wants to exercise its right to vote, it should recall the stock in good time so that a proxy voting form can be completed and returned to the registrar by the required deadline.

Open Borrow vs. Guaranteed Borrow

In general, a stock lending and borrowing transaction has no fixed maturity – an **open maturity**, and either party can terminate the transaction early on demand. This transaction is called an "**open borrow**".

Sometimes, a stock lending and borrowing transaction has a **fixed maturity** and neither party can terminate the transaction early, unless agreement by both parties. In other words, the stock lender guarantees the availability of the stock to the stock borrower during the term of the transaction. This transaction is called a "**guaranteed borrow**".

1.3.2 Stock Lending/Borrowing Transaction Flows

Flows at Inception

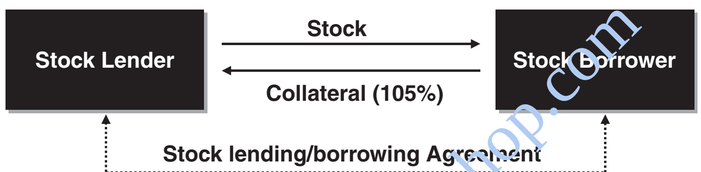

Figure 1.16 Stock lending/borrowing flows at inception.

At inception, three processes take place (as shown in Figure 1.16):

- The securities lending and borrowing agreement is agreed and signed by the two counterparties.
- The stock is transferred from the lender custody account to the borrower custody account. Legal title passes to the borrower, so he/she can sell or on-lend the stock.
- Collateral is posted by the borrower. Loaned stock is generally collateralized, reducing the lender's credit exposure to the borrower. Commonly, the borrower can post cash or other liquid securities. The market value of the collateral posted is typically 105% of the market value of the stock borrowed.

Flows during the Life of the Stock Lending and Borrowing Agreement

During the life of the stock lending and borrowing agreement, the following processes take place (as shown in Figure 1.17):

- The borrower pays a lending fee – the borrowing fee – to the lender.
- The lender pays interest to the borrower on the cash collateral. The interest is commonly calculated on a daily basis.

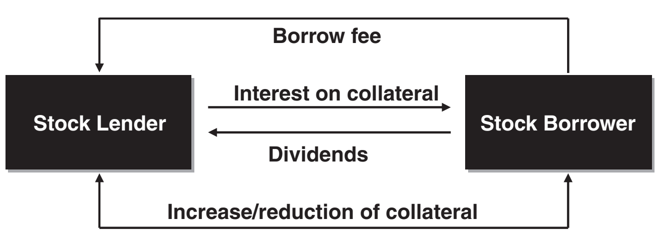

Figure 1.17 Stock lending/borrowing flows during the life of the agreement.

- The borrower pays the lender an amount equivalent to the dividends distributed to the borrowed shares. This amount is called the "**manufactured dividend**".
- Collateral is readjusted on a daily basis, so the market value of the collateral posted is 105% of the market value of the stock borrowed.

Flows at Maturity or Upon Early Termination

At maturity of the agreement, or upon early termination, the following processes take place (as shown in Figure 1.18):

- The stock is returned from the borrower custody account to the lender custody account.
- Collateral is returned by the lender to the borrower.

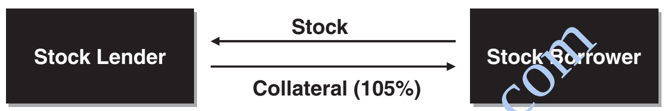

Figure 1.18 Stock lending/borrowing flows at maturity (or early termination).

1.3.3 Counterparty Credit Risk

Following an event of default, the stock lending and borrowing agreement provide for the non-defaulting counterparty to calculate its exposure to the defaulting counterparty. From a stock lender point of view, the net exposure is calculated as the cost of repurchasing the lent securities less the funds raised from the sale of the collateral. A deficit represents a claim on the defaulting counterparty in administration.

The stock lender is credit exposed to the stock borrower. Following an event of default, the non-defaulting stock lender has no claim on the defaulting stock borrower to return the lent securities as securities lending/borrowing is legally a transfer of ownership. However, in the event that the borrower defaults, the stock lender would set off the market value of the collateral against the market value of the lent stock. The lender owns the collateral and can either sell it, in the case of collateral other than cash, or use it, in the case of cash collateral, in order to repurchase the lent stock. This way, the stock lender is restored to its original position of owning the lent stock. There are several events that trigger the credit risk exposure to the borrower:

- During the life of the agreement, if the stock significantly increases in value, the stock borrower is required to post additional collateral and the borrower may fail to meet this obligation.
- During the life of the agreement, the stock borrower may fail to pay the manufactured dividend.
- During the life of the agreement, the stock borrower may fail to pay the borrow fee.
- At maturity, or upon early termination, the borrower may fail to return the stock. It would oblige the stock lender to buy back the lent stock in the market using the posted collateral. This collateral may not be sufficient to pay for the stock.

To reduce the credit risk exposure to the stock borrower, "**haircuts**" are employed. In other words, the stock borrower is required to provide collateral with a market value greater than the market value of the stock borrowed. Typically the haircut is 5% of the market value of the borrowed/lent stock (i.e., the collateral posted is 105% of the stock market value). Additionally, most stock-lending transactions are agreed on an open borrow basis, meaning that either counterparty can cancel the transaction with a short notice period. In a deteriorating credit environment, the stock lender can reduce his/her exposure in very short order.

The stock borrower is also credit exposed to the stock lender. There are several events that trigger the credit risk exposure to the lender:

- During the life of the agreement the stock lender may become insolvent, failing to pay the interest on the cash collateral.
- During the life of the agreement the stock lender may fail to return collateral in the case of a fall in value of the borrowed stock.
- At maturity, the stock lender may fail to return collateral. If the lender's insolvency takes place immediately after a sharp decline of the stock, the collateral posted may well exceed the market value of the stock, generating a loss for the stock borrower.

The credit risk to the stock lender may be very relevant if the lender is a major owner of the stock. The stock lender's insolvency may trigger a widespread sale of the stock, increasing the probability of a large loss for the stock borrower. However, most stock lending transactions are agreed on an open borrow basis, meaning that either counterparty can cancel the transaction with a short notice period. In a deteriorating credit environment, the stock borrower can reduce his/her exposure in very short order.

1.3.4 Advantages of Stock Lending and Borrowing

There are many positive aspects of stock lending and borrowing, for example:

- It allows stock investors to earn additional income by lending their stock on to third parties, enhancing the return of their stock portfolios.
- It allows stock investors to raise financing when the stock is lent against cash collateral.
- It supports many trading and risk management strategies that otherwise would be extremely difficult to execute.
- It increases the liquidity of the securities market by allowing securities to be borrowed temporarily; thus reducing the potential for failed settlements and their associated penalties.
- It is easy to implement and at a low cost. There are no depository or transaction costs on the lent stocks during the life of the transaction.
- It allows for early termination. There is the possibility for both counterparties to end the transaction at any time if the maturity is on an open basis.

1.3.5 Drawbacks of Stock Lending and Borrowing

There are some negative aspects of stock lending and borrowing, for example:

- The stock lender losses the voting rights associated with the lent shares.
- The stock lender is credit exposed to the stock borrower.
- The stock borrower is credit exposed to the stock lender.
- It may add selling pressure to the market on illiquid stocks by assisting stock short sellers.
- It may increase tax authorities' scrutiny of the investor as stock lending and borrowing transactions have sometimes been used to implement tax arbitrage schemes.

1.4 CALL AND PUT OPTIONS

In this section I will cover plain vanilla options. A vanilla option, also called a standard option, is a call or put in its most basic form. Options are a means for their buyers to gain either long or short exposure to an equity underlying with a limited downside.

1.4.1 Call Options

Call options allow an investor to take bullish views on an underlying stock, a basket of stocks or a stock index.

- A physically settled European call option provides the buyer – the holder – the right, but not the obligation, to buy a specified number of shares of an equity underlying at a predetermined price – the strike price – at a future date – the expiration date. In return for this right, the buyer pays an upfront premium for the call.
- A cash-settled European call option provides the buyer the appreciation of a specified number of shares of an equity underlying above a predetermined price – the strike price – at a future date – the expiration date. The buyer pays an upfront premium for the call.

At expiry, the holder of the call will exercise the option if the share price of the underlying stock is higher than the strike price. Thus, if the share price ends up lower than the strike price, the holder will not exercise the call. The holder has unlimited upside potential, while his/her loss is limited to the option premium paid.

As an example, let us assume that on 3 June 20X1, ABC is looking to buy IBM stock in six months' time. ABC believes that the stock of IBM will significantly increase in value over the next six months and acquires from Gigabank a European call option on one million IBM shares. On 3 June 20X1, IBM stock is trading at USD 150. The physically settled call option has the following terms:

Physically Settled Call Option – Main Terms	
Buyer	ABC Corp
Seller	Gigabank
Option type	Call
Trade date	3-June-20X1
Expiration date	3-December-20X1
Option style	European
Shares	IBM
Number of options	1 million
Option entitlement	One share per option
Strike price	USD 180.00 (120% of the spot price)
Spot price	USD 150.00
Premium	2.66% of the notional amount USD 4 million (i.e., USD 4 per share)
Premium payment date	Two currency business days after the trade date (5-June-20X1)
Notional amount	Number of options × Spot price USD 150 million
Settlement method	Physical settlement
Settlement date	6-December-20X1 (three exchange business days after the expiration date)

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By buying the call option ABC has the right, but not the obligation, to buy on the "settlement date" one million shares of IBM at a strike price of USD 180 per share. Because upon exercise ABC would be buying the underlying stock, the call is a physically settled call. ABC pays to Gigabank a premium of USD 4 million on 5 June 20X1. Because it is European-style, the option can only be exercised at expiry. On the expiration date, 3 December 20X1, ABC would be assessing whether to exercise the option, as follows:

- If IBM's stock price is greater than the USD 180 strike price, ABC would exercise the call. On the "settlement date" ABC would receive from Gigabank one million shares of IBM in exchange for USD 180 million. For example, if at expiry IBM stock is trading at USD 210, ABC would exercise the call option paying USD 180 per share for a stock worth USD 210 per share.
- If IBM's stock price is lower than or equal to the USD 180 strike price, ABC would not exercise the option.

In a similar example, let us assume that ABC is not interested in having the right to buy one million shares of IBM but instead in receiving the appreciation of one million shares of IBM above USD 180. ABC then buys a cash-settled European call option on IBM stock with the following terms:

Cash-settled Call Option – Main Terms	
Buyer	ABC Corp
Seller	Gigabank
Option type	Call
Trade date	3-June-20X1
Expiration date	3-December-20X1
Option style	European
Shares	IBM
Number of options	1 million
Option entitlement	One share per option
Strike price	USD 180.00 (120% of the spot price)
Spot price	USD 150.00
Premium	2.66% of the notional amount USD 4 million (i.e., USD 4 per share)
Premium payment date	5-June-20X1 (two currency business days after the trade date)
Notional amount	Number of options × Spot price USD 150 million
Automatic exercise	Applicable
Settlement price	The closing price of the shares on the valuation date
Settlement method	Cash settlement
Cash settlement amount	The maximum of: (i) Number of options × (Settlement price – Strike price), and (ii) Zero
Cash settlement payment date	6-December-20X1 (three exchange business days after the expiration date)

ABC pays, on 5 June 20X1, a USD 4 million premium. On expiration date, ABC will exercise the call if IBM's stock price – the settlement price – is above the USD 180 strike price. What if ABC forgets to exercise the call? The contract includes a term, "automatic exercise", which prevents the buyer from forgetting to exercise an in-the-money option. In our option, the "automatic exercise" term is defined as "applicable", meaning that if the option is in-the-money on expiration date, it would automatically be exercised. More precisely, on 6 December 20X1 ABC would be receiving the "cash settlement amount". This amount is calculated as follows:

- If the "settlement price" is greater than the USD 180 strike price, the option would be exercised and ABC would receive an amount equivalent to Number of options × (Settlement price – Strike price) = 1 million × (Settlement price – 180). In other words, ABC receives from Gigabank the appreciation of the shares above USD 180. For example, if at expiry IBM stock price has risen to USD 210, the call would be exercised and ABC would receive from Gigabank USD 30 million [= 1 million shares × (210 – 180)]. Taking into account the USD 4 million initial premium paid, the overall payoff for ABC would be a profit of USD 26 million (= 30 million – 4 million).
- If the settlement price is lower than or equal to the USD 180 strike price, the cash settlement amount would be zero. The option would not be exercised and, thus, ABC would receive nothing. Taking into account the USD 4 million initial premium paid, the overall payoff for ABC would be a loss of USD 4 million.

Options strategies are often described using "payoff" graphs which show the value of an option (i.e., the cash settlement amount) on the expiration date after subtracting the upfront premium. Figure 1.19 shows the payoff for ABC under the IBM call. Note that in the graph the USD 4 million option premium has been taken into account ignoring timing differences. In reality, the premium is paid upfront while the payout of the option is received shortly after the option expiration date.

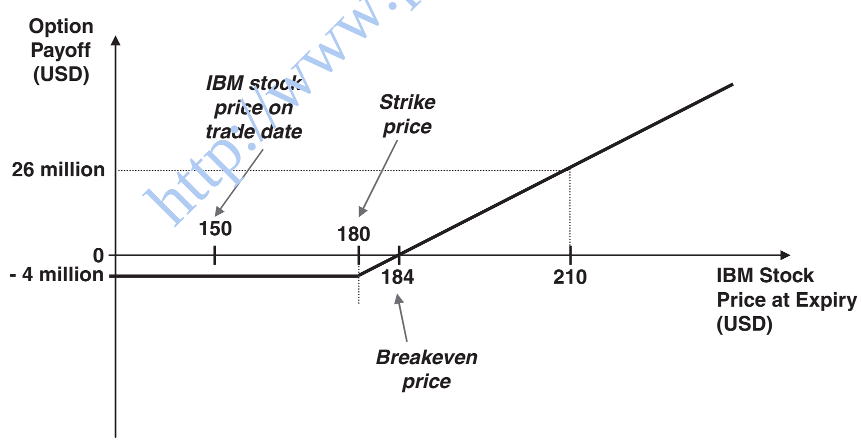

Figure 1.19 Payoff to the buyer of the call option.

The graph shows that there is a positive payoff for ABC, the option buyer, if the stock price at expiration is greater than the USD 184 breakeven price. The breakeven price is calculated as the sum of the USD 4 per share call premium and the USD 180.00 strike. By the same reasoning, there is a negative payoff if the stock price at expiration is lower than the breakeven price. The graph also shows that for a buyer of a call the upside is unlimited while the downside is limited to the initial premium paid.

Conversely, the seller of the IBM call, Gigabank, has a positive payoff if the stock price at expiration is lower than the breakeven price (see Figure 1.20). Applying the same reasoning,

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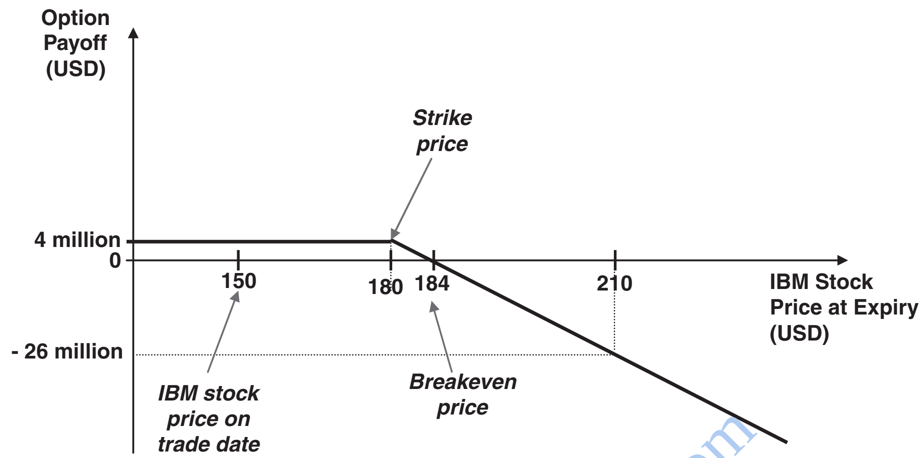

Figure 1.20 Payoff to the seller of the call option.

there is a negative payoff for the seller of the option if the stock price at expiry is greater than the breakeven price. The graph also shows that for a seller of a call the upside is limited to the initial premium received, while there is an unlimited downside.

1.4.2 Put Options

Put options allow an investor to take bearish views on an underlying stock.

- A physically settled European put option provides the buyer – the holder – the right, but not the obligation, to sell a specified number of shares at a predetermined price – the strike price – at a future date – the expiration date. In return for this right, the buyer pays an upfront premium for the put.
- A cash-settled European put option provides the buyer the depreciation of a specified number of shares below a predetermined price – the strike price – at a future date – the expiration date. The buyer pays an upfront premium for the put.

As an example, let us assume that on 3 June 20X1 ABC has a bearish view on IBM stock. ABC believes that IBM stock price will significantly fall over the next six months and acquires from Gigabank a European put option on one million IBM shares. On 3 June 20X1, IBM stock is trading at USD 150. Let us assume further that ABC is not interested in having the right to sell one million shares of IBM but instead in having the right to receive the depreciation of IBM's stock below USD 120. The cash-settled put option has the following terms:

Cash-settled Put Option – Main Terms	
Buyer	ABC Corp
Seller	Gigabank
Option type	Put
Trade date	3-June-20X1
Expiration date	3-December-20X1
Option style	European

Cash-settled Put Option – Main Terms	
Shares	IBM
Number of options	1 million
Option entitlement	One share per option
Strike price	USD 120.00 (80% of the spot price)
Spot price	USD 150.00
Premium	1.33% of the notional amount USD 2 million (i.e., USD 2 per share)
Premium payment date	5-June-20X1 (two currency business days after the trade date)
Notional amount	Number of options × Spot price USD 150 million
Settlement price	The closing price of the shares on the valuation date
Settlement method	Cash settlement
Cash settlement amount	The maximum of: (i) Number of options × (Strike price – Settlement price), and (ii) Zero
Cash settlement payment date	6-December-20X1 (three exchange business days after the expiration date)

ABC pays, on 5 June 20X1, a USD 2 million premium. At expiry, the holder of the put – ABC – will exercise the option if IBM's stock price is lower than the USD 120 strike price. Putting it in a more formal way, on the "cash settlement payment date" (6 December 20X1) ABC would be receiving the "cash settlement amount". This amount is calculated as follows:

- If the settlement price is lower than the USD 120 strike price, the option would be exercised and ABC would receive an amount equivalent to Number of options × (Strike price – Settlement price) = 1 million × (120 – Settlement price). In other words, ABC would receive from Gigabank the depreciation of the shares below USD 120. For example, if at expiry IBM stock price has fallen to USD 110, ABC would exercise the put, receiving from Gigabank USD 10 million [= 1 million shares × (120 – 110)]. Taking into account the USD 2 million initial premium paid, the overall payoff for ABC would be a profit of USD 8 million (= 10 million – 2 million).
- If the "settlement price" is greater than or equal to the USD 120 strike price, the option would not be exercised and ABC would receive nothing as the cash settlement amount would be zero. Taking into account the USD 2 million initial premium paid, the overall payoff for ABC would be a loss of USD 2 million.

Figure 1.21 shows the payoff for ABC under the IBM put. The graph illustrates the value of the option (i.e., the cash settlement amount) on the expiration date after subtracting the USD 2 million upfront premium. Note that in the graph the option premium has been taken into account ignoring timing differences. In reality, the premium is paid upfront and the payout of the option is received shortly after the option's expiration date.

The graph shows that there is a positive payoff for the option buyer if the stock price at expiry is lower than the USD 118 breakeven price. The breakeven price is calculated as the USD 120 strike minus the USD 2 per share premium. Applying the same reasoning, there is a negative payoff if the stock price at expiry is greater than the USD 118 breakeven price. The graph also shows that the upside is limited while the downside is limited to the initial premium

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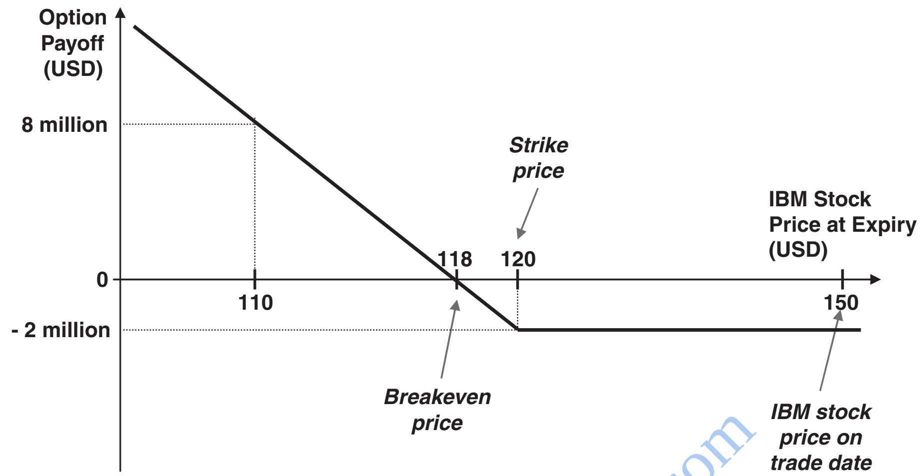

Figure 1.21 Payoff to the buyer of the put option.

paid. In our example, the maximum upside for ABC is USD 118 million (= 120 million – 2 million), reached when IBM stock price trades at zero on the expiration date.

The seller of the IBM put, Gigabank, has a positive payoff if the stock price at expiry is greater than the USD 118 breakeven price (see Figure 1.22). Applying the same reasoning, there is a negative payoff for the seller of the option if the stock price at expiry is lower than the USD 118 breakeven price. The graph also shows that the upside is limited to the initial premium received, while there is a limited downside. The maximum downside for Gigabank is USD 118 million (= 120 million – 2 million), reached if IBM stock price trades at zero on the expiration date.

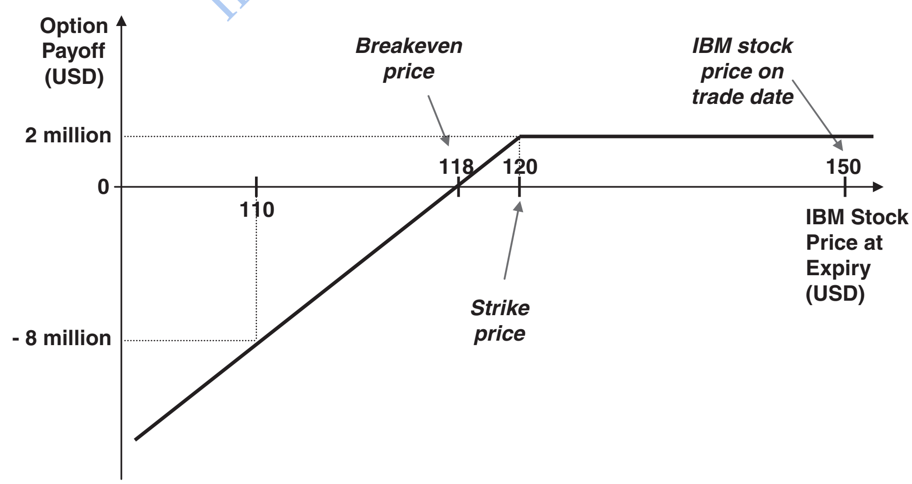

Figure 1.22 Payoff to the seller of the put option.

1.4.3 European vs. American Style

If an option can only be exercised on the expiry date it is called a **European-style** option. If an option can be exercised at any time up to the expiry date it is called an **American-style** option. Options can have more elaborate exercise schedules.

1.4.4 Time Value vs. Intrinsic Value

The value of an option before expiry is comprised of the sum of two components: its intrinsic value and its time value.

$$\text{Total value} = \text{Intrinsic value} + \text{Time value}$$

The intrinsic value of an option is the value it would have if it were exercised immediately. The intrinsic value of a call option is calculated as follows:

- When the stock price is above the strike price, the call option is said to have intrinsic value. This is because, were the call to expire at that moment there would be a positive cash payout.
- When the stock price is below or at the strike price, the call option is said to have no intrinsic value. This is because, were the call to expire at that moment there would be no cash payout.

$$\text{Call intrinsic value} = \text{Number of options} \times \text{Max}[(\text{Stock price} - \text{Strike price}), 0]$$

The intrinsic value of a put option is calculated as follows:

- When the stock price is below the strike price, the put is said to have intrinsic value. This is because, were the put to expire at that moment there would be a positive cash payout.
- When the stock price is above or at the strike price, the put is said to have no intrinsic value. This is because, were the put to expire at that moment there would be no cash payout.

$$\text{Put intrinsic value} = \text{Number of options} \times \text{Max}[(\text{Strike price} - \text{Stock price}), 0]$$

The time value of an option is the portion of the value of an option that is due to the fact that it has some time to expiration. The time value of an option represents the possibility that the option may finish in-the-money or further in-the-money. The time value will progressively erode as the option approaches its expiration date. At expiry there will be no time value. The time value component is calculated as the difference between the total value of an option and its intrinsic value:

$$\text{Time value} = \text{Total value} - \text{Intrinsic value}$$

Figure 1.23 illustrates the intrinsic value and time value components of a call option on 1 million IBM shares, a USD 180 strike and 6 months to expiration, assuming a 4% interest rate, a 2% dividend yield and a 30% implied volatility (note that the y-axis has not been graphed using a linear scale to better highlight the concepts). The total value of the option has been calculated using an option pricing model. For example, assuming IBM's spot price at USD 210, the total value of the call option would be USD 37 million. The intrinsic value

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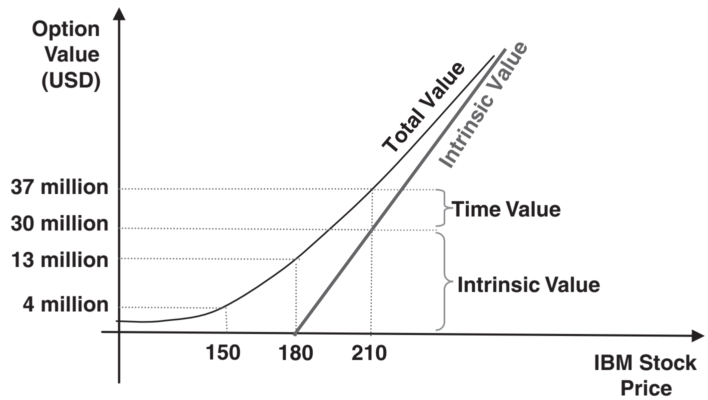

Figure 1.23 Intrinsic value vs. time value of a call option.

would be USD 30 million [= 1 million × (210 – 180)]. Therefore, the option time value would be USD 7 million (= 37 million – 30 million). The following table summarizes the intrinsic value and time value components for three stock price scenarios:

Spot price	USD 150	USD 180	USD 210
Intrinsic value	0	0	USD 30 million
Time value	USD 4 million	USD 13 million	USD 7 million
Total value	USD 4 million	USD 13 million	USD 37 million

1.4.5 In, At or Out-of-the-money

Options which have intrinsic value are described as being "**in-the-money**". By the same reasoning, options which have no intrinsic value (e.g., in a call option, if the share price is below the strike price) are called "**out-of-the-money**". If the option expires out-of-the-money, the holder will not exercise the option. An option is called "**at-the-money**" if the stock price is at the strike price.

Description	Call	Put	Intrinsic value
In-the-money	Stock price > Strike	Stock price < Strike	Yes
At-the-money	Stock price = Strike	Stock price = Strike	No
Out-of-the-money	Stock price < Strike	Stock price > Strike	No

Based on our previous call option on IBM with a strike price of USD 150:

Spot price	USD 150	USD 180	USD 210
Strike	USD 180	USD 180	USD 180
Moneyness	Out-of-the-money	At-the-money	In-the-money

At expiry, there will be no time value and there will be two different scenarios:

- The option expires in-the-money and there is a positive cash payout; or
- The option expires out-of-the-money and is worthless.

1.4.6 Variables that Influence an Option Price

Option prices are calculated using mathematical models. Most banks use in-house variations of the Black–Scholes model. In this section, I will not cover option pricing models as there are plenty of excellent books on the subject. However, I think it is important to understand how options are affected by the variables that drive option prices. Let us assume that we are already long a call or a put option. Ongoing valuation of the option depends on the following seven variables (see Figure 1.24):

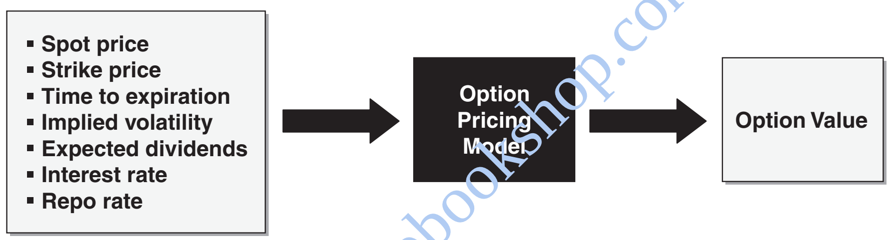

Figure 1.24 Variables that affect an option valuation.

1. The **spot price**. It is the current price of the underlying stock. In the case of a call, as the stock price rises the value of the call increases. In the case of a put, as the stock price falls the value of the put increases, assuming everything else is equal.

	Call	Put
Spot price increases	↑	↓
Spot price decreases	↓	↑

2. The **strike price**. It is the exercise price of the option. It can be expressed as either an absolute level or a percentage of the spot price. Market participants commonly use the latter. A higher strike price means a lower call value, and a lower strike price means a higher call value. A higher strike price means a higher put value, and a lower strike price means a lower put value.

3. The **time to expiration**. It is the time period remaining to expiration. It is calculated as the difference, commonly in days, between the expiration date and the current date. Generally, the longer the time until expiration, the greater the value of an option. In other words, the longer the time until expiration, the higher the probability that the option will expire with a larger positive value. Note that sometimes changes in time to expiration have an ambiguous effect on European options due to the potential adverse effect of dividend payments during the life of the option.

	Call	Put
Time to expiration increases	↑	↑
Time to expiration decreases	↓	↓

4. The **implied volatility**. It is the expected volatility of the underlying during the life of the option. It is an estimation of the amplitude in the movement of the underlying stock price. Because the option payoff is asymmetric (i.e., the maximum payoff is larger than the premium paid), the higher the volatility, the more likely that a large movement of the stock price will translate into a larger payoff. Therefore, the higher the implied volatility, the more expensive is the option. The implied volatility is an annualized statistic expressed in percentage terms.

	Call	Put
Implied volatility increases	↑	↑
Implied volatility decreases	↓	↓

5. The **expected dividends** to be distributed to the underlying shares. If the company of the underlying stock decides to increase the dividends to be paid during the life of the option, this will cause a larger than expected drop in the stock price once it goes ex-dividend. Higher dividends lower the forward price, thus reducing the price of the call and increasing the price of the put.

	Call	Put
Expected dividends increase	↓	↑
Expected dividends decrease	↑	↓

6. The **interest rate**. Purchasing a call is comparable to buying a portion of the underlying stock. In a perfect market and in absence of dividends, the stock is expected to appreciate identically to an investment in a riskless interest rate instrument. As a result, an increase in interest rates implies that the underlying stock will rise higher. A call price will increase as the interest rate increases. Similarly, a put price will decrease as the interest rate increases.

	Call	Put
Interest rate increases	↑	↓
Interest rate decreases	↓	↑

7. The **repo rate**. It is the borrowing fee paid by a stock borrower of the underlying shares. The higher the repo rate, the lower the forward price. As a result, the higher the repo rate, the lower the price of the call and the higher the value of a put.

	Call	Put
Repo rate increases	↓	↑
Repo rate decreases	↑	↓

The following table summarizes the impact on the value of a call and a put on a specific stock of an increase in the specified variable, assuming everything else is equal. It can be observed that changes in time to expiration and volatility have similar effects on the call and put values, while changes to the other factors affect call and put values in an opposite way.

Variable increased	Effect in call value	Effect in put value
Spot price	↑	↓
Strike price	↓	↑
Time to expiration	↑	↑
Implied volatility	↑	↑
Dividend	↓	↑
Interest rate	↑	↓
Repo	↓	↑

1.4.7 Historical Volatility vs. Implied Volatility

Historical volatility, or **realized volatility**, is the annualized standard deviation of the logarithm of price returns of an underlying over a specific period of time. It measures the historical variations of the underlying price over a certain time. It is retrospective, based on historical data, commonly using the underlying closing prices. It is expressed as an annualized percentage.

Implied volatility is the expected future realized volatility of the underlying during the life of the option. It is expressed as an annualized percentage.

The historical volatility is calculated as follows:

$$\text{Historical volatility} = \sigma_H = 100 \times \sqrt{\frac{252 \times \sum_{i=1}^{N} \left(Ln \frac{P_i}{P_{i-1}} \right)^2}{N}}$$

where:

Ln is the natural logarithm.

P_i and P_{i-1} are the official levels of the underlying on respectively the ith and i–1th observation days. In most cases the official level is the daily closing price.

N is the number of days that, as of the trade date, are expected to be scheduled trading days for the period from, but excluding the observation start date to, and including, the observation end date.

Observation day is each trading day during the observation period.

Observation period is the period from, but excluding, the observation start date to, but excluding, the observation end date.

As an example, let us take the stock prices of a stock during a month to calculate the historical volatility performing the following steps:

1. Calculate the daily lognormal returns.
2. Square each return to capture size, but not sign.
3. Sum the squared returns over the period.
4. Annualize by the 252 trading days per year.

5. Divide by the number of observations.
6. Take the square root to convert variance to volatility.
7. Multiply by 100 to express it as a percentage.

Day	Stock price	$Ln(P_i/P_{i-1})$	$[Ln(P_i/P_{i-1})]^2$
1	123		
2	117	−0.05001	0.002501
3	114	−0.02598	0.000675
4	111	−0.02667	0.000711
5	113	0.017858	0.000319
6	116	0.026202	0.000687
7	119	0.025533	0.000652
8	112	−0.06062	0.003675
9	118	0.052186	0.002723
10	123	0.0415	0.001722
11	117	−0.05001	0.002501
12	119	0.01695	0.000287
13	120	0.008368	0.000070
14	119	−0.00837	0.000070
15	124	0.041158	0.001694
16	125	0.008032	0.0000645
17	123	−0.01613	0.00026
18	122	−0.00816	0.0000666
19	121	−0.00823	0.0000677
20	120	−0.0083	0.0000689
		Sum	0.018816

The sum of the squared returns over the period was 0.018816. The annualization of this figure was 4.741632 (= 0.018816 × 252). The adjustment of the annualized figure by dividing by the number of observations was 0.2371 (= 4.741632/20). The historical volatility in percentage terms was 23.71% (= 0.2371 × 100).

1.4.8 Put–Call Parity

The prices of European-style calls and puts on the same stock, with the same strike and the same time to expiration, are related by an expression termed "**put–call parity**" as follows:

$$S + P = \text{PV}(K) + C + \text{PV}(\text{Dividends}) + \text{PV}(\text{Repo})$$

where S is the stock's spot price, P is the put price, PV(K) is the present value of the exercise price, C is the call price, PV(Dividends) is the present value of the expected dividends and PV(Repo) is the present value of the stock borrowing fee.

The relationship can be understood by considering two portfolios; the first is comprised of the stock and the put option, and the second is comprised of the call and cash equal to the present value of the exercise price.

- If at expiry the stock price is above the strike, the put will not be exercised and the first portfolio would be worth the stock price at expiry. In the second portfolio, the call will be exercised, receiving the underlying stock in exchange for cash worth the strike. As a result, the second portfolio would be worth the stock price at expiry.

- If at expiry the stock price is below the strike, the put will be exercised, receiving cash worth the strike and delivering the underlying stock. As a result, the first portfolio would be worth the strike at expiry. In the second portfolio, the call will not be exercised and the cash would be worth the strike. As a result, the second portfolio would be worth the strike at expiry.

At expiry, both portfolios are worth the greater of the stock price and the strike price. As both portfolios are worth the same at expiry, then at inception of the strategy they must be worth the same too.

1.4.9 Options' Sensitivities, the "Greeks"

The essential risk measures of an option value are known as the "Greeks". An option sensitivity to each variable is measured by taking the partial derivative of the option value with respect to that particular variable. Option traders measure option Greeks in order to risk manage their option positions. In this subsection I will briefly cover the four so-called "first-order" Greeks and "gamma", which is a second-order Greek (see table below). A first-order Greek represents a first-order derivative of the option value with respect to a specific variable.

Variable	Symbol	Measures option sensitivity to
Delta	Δ	Change in underlying price
Vega	ν	Change in volatility
Gamma	Γ	Change in delta
Theta	None	Change in time to expiration
Rho	None	Change in interest rates

Delta

An option delta indicates the theoretical change in an option price with respect to changes in the price of the underlying stock. When the stock price changes by a small amount, the option price changes by the delta multiplied by that amount. The delta is commonly expressed as a percentage, measuring the change in an option price for a 1% change in the underlying stock price.

Figure 1.25 shows the delta of a European call on IBM stock with a strike price of USD 180, a time to expiration of 6 months, an interest rate of 4%, a dividend yield of 2% and a volatility of 30%, for different levels of IBM's stock price. It can be observed that the delta of a call option is always positive, as the call price increases when the underlying stock price rises. The graph shows that at USD 176 (a stock price quite close to the strike price), the delta of the call option is 50%. If the stock price moves up by USD 1 to USD 177, the call option would see its value increase by approximately USD 0.50 (= USD 1 $\times$ 50%).

Figure 1.26 shows the delta of a European put on IBM stock with a strike price of USD 180, a time to expiration of 6 months, an interest rate of 4%, a dividend yield of 2% and a volatility of 30%, for different levels of IBM's stock price. It can be observed that the delta of a put option is always negative as a rise in the underlying stock price would cause the put value to decrease. In other words, the put price increases when the underlying falls.

The absolute value of the delta can be loosely interpreted as an approximate measure of the probability that an option will expire in-the-money. If an option is very deep in-the-money, and therefore has a very high probability of being in-the-money at expiry, the absolute value of the delta will be close to 100%. If an option is very deep out-of-the-money, it has a low

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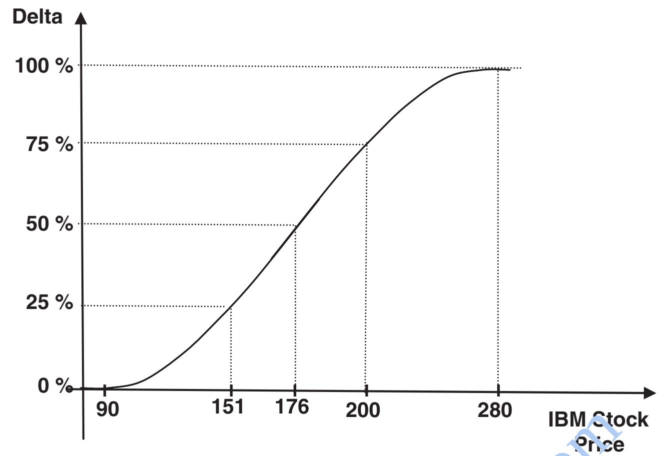

Figure 1.25 Delta of a European call as a function of IBM's stock price.

probability of being in-the-money at expiry, and therefore the absolute value of its delta will be close to zero. At-the-money options have a delta close to 50%, meaning roughly a 50% probability of being exercised at expiry.

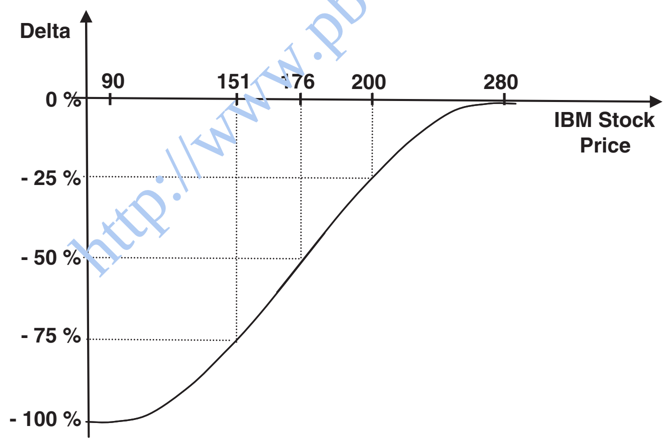

Figure 1.26 Delta of a European put as a function of IBM's stock price.

Relationship between call and put deltas

Given a call and put option on the same underlying, strike price and time to expiration, the sum of the absolute values of the delta of each option is always 100%. Therefore, if the delta for an option is known, one can compute the delta of an option with the same strike price, underlying and maturity but opposite right by subtracting 100% from the known value. For example, if the delta of a call is 35% then one can compute the delta of the corresponding put with the same strike price as 35% – 100% = –65%.

$$\text{Call delta} + \text{Absolute value(Put delta)} = 100\%$$

Vega

Vega measures the sensitivity of an option value to small changes in the implied volatility of the underlying. Note that vega is not actually the name of any Greek letter. Vega is typically expressed as the amount of money, per underlying share, the option value will gain or lose as volatility rises or falls by 1%. Vega can be an important Greek to monitor for an option trader, especially in volatile markets since some of the value of option strategies can be particularly sensitive to changes in volatility.

Vega increases as time to expiration increases. Vega is usually larger for at-the-money options. Vega generally increases with implied volatility, assuming everything else is equal.

Gamma

It was observed in the delta section that delta is a non-linear function. This is due to gamma. Gamma measures the sensitivity of an option delta to small changes in the underlying stock price. When a trader seeks to establish an effective delta-hedge for an option, he/she may also seek to neutralize the option gamma, as this will ensure that the hedge will be effective over a wider range of underlying price movements. Deep in-the-money or deep out-of-the-money options tend to have low gamma for both calls and puts. The value of gamma is higher for at-the-money, short-dated options. For example, in our 6-month call option on IBM at USD 180 stock price, the gamma of the option is 1.85%. Therefore, for a 1% upward move in IBM's stock price, the option delta will increase by 1.85%.

Theta

Theta measures the sensitivity of the option value to small changes in the time remaining to expiration. Theta is usually defined as the change in the value of an option for a one-day decrease in the time to expiration. Theta is always negative for both call and put options. The decrease in the value of calls and puts as time passes is known as time decay. Theta is non-linear, meaning that its value decreases faster as the option approaches maturity. For at-the-money options, where the value of the underlying is close to the strike price, the time decay increases as the option approaches expiration. For in-the-money and out-of-the-money options, where the underlying is not close to the strike price, the time decay is more linear.

Rho

Rho measures the sensitivity of the option value to small changes in the applicable interest rate. The changes in option prices for changes in interest rates are relatively small. For this reason, rho is the least used of the first-order Greeks. Rho is positive for call options and negative for put options.

1.4.10 Delta Hedging

As covered earlier, delta measures the sensitivity of an option value to small changes in the share price of the underlying stock. An option trading desk managing the position in an option is usually not interested in being exposed to directional movements in the underlying stock. The desk role is to profit from changes in implied and realized volatility. The trading desk will typically delta-hedge the option position so that the overall delta is zero. In this way a position in an option has less exposure to directional movements in the underlying stock, leaving exposure to changes in implied and realized volatility. Because the option delta changes with market conditions and time to expiry, the hedge needs to be adjusted continuously. Delta hedging is the term given to the process of ensuring that an option position has little or no exposure to the movements of the underlying stock price.

1.4.11 Offsetting Dividend Risk

When pricing a call or a put option, a string of expected dividends has to be assumed. Sometimes it is difficult to forecast the expected dividends, and banks may be unwilling to take dividend risk unless a very conservative dividend forecast is assumed.

Adjustments to Strike and Notional

An alternative is to assume a string of expected dividends, and adjust the option terms for any deviation of the realized dividends relative to the forecasted dividends. For long-term options on single stocks, it is a good way to improve their pricing. It means that the client, and not the bank, is taking the dividend risk. Commonly, the strike and notional are adjusted on the ex-dividend date for any deviation of the paid dividends relative to the assumed dividends, as follows:

Adjusted strike = Strike × Adjustment factor
Adjusted number of options = Number of options/Adjustment factor
Adjustment factor = (Record share price − Dividend deviation)/Record share price
Dividend deviation = Gross dividend − Assumed dividend

The "Record share price" is typically the closing share price on the day immediately preceding the ex-dividend date. Therefore, if the underlying stock distributes a gross dividend larger than the assumed dividend, the strike would be adjusted downwards, while the number of shares would be adjusted upwards.

Payment of the Dividend Adjusted for the Delta

Sometimes the counterparties to an option agree on a specific dividend string but, in case of a deviation of real dividends relative to assumed dividends, they prefer not to adjust the terms of the option. This could be the case of a transaction that involves a financing backed by a stock and a put to protect the value of the stock. A downwards adjustment of the put strike would probably imply the adjustment to the terms of the financing, usually a cumbersome task.

An alternative to adjusting the terms of the option is to make a payment that compensates the dividend deviation. The compensation amount is calculated as:

$$\text{Compensation amount} = \text{Delta} \times \text{Number of options} \times \text{Dividend deviation}$$

where:

$$\text{Dividend deviation} = \text{Realized gross dividend} - \text{Assumed dividend}$$

The delta is the delta of the option on the dividend record date (i.e., the trading day prior to the ex-dividend date). The delta of an option is not an absolutely transparent figure, depending on the model used. The delta is usually calculated by the bank at its sole discretion, but the other counterparty may challenge the calculation.

As an example, let us assume that the parties to an option agreed on a EUR 0.20 dividend for a specific semiannual period, and that during this period the distributed gross dividend was EUR 0.30. Let us assume further that the option is a call on 10 million shares and that the delta on the closing of the record date was 40%. The compensation amount would be calculated as follows:

$$\text{Compensation amount} = 40\% \times (10 \text{ million options}) \times (0.30 - 0.20) = \text{EUR } 400{,}000$$

Now we need to decide which counterparty to the option is required to pay this amount to the other party. Let us assume that the buyer – e.g., the bank – of the call option delta-hedges its position. The buyer would need to be short 4 million shares (= 40% delta × 10 million options). In other words, the buyer of the call would need to borrow 4 million shares of the underlying stock. On the dividend payment date, and under the stock borrowing/lending agreement, the bank would be required to pay EUR 1.2 million (= 0.30 dividend × 4 million borrowed shares) to the stock lender. However, the bank assumed that it was going to pay only EUR 0.8 million (= 0.20 dividend × 4 million borrowed shares) when it priced the option at its inception. As a result, the bank would need to receive EUR 400,000 from the other party to the option. Therefore, if the compensation amount is positive (i.e., a distributed dividend greater than the assumed dividend), this amount should be received by the call option buyer – the bank in this example – from the call option seller – the client in this example. Conversely, if the compensation amount is negative (i.e., a distributed dividend lower than the assumed dividend), the call option buyer must pay to the option seller the absolute value of the compensation amount.

In the case of a put option, the compensation amount is calculated in a similar way. However, if the compensation amount is positive (i.e., a distributed dividend greater than the assumed dividend), this amount should be paid by the put option buyer to the put option seller. Conversely, if the compensation amount is negative (i.e., a distributed dividend lower than the assumed dividend), the put option seller must pay to the option buyer the absolute value of the compensation amount. The following table summarizes the compensation amount payer/receiver for each type of option:

Dividend deviation	Call option – compensation amount payer	Put option – compensation amount payer
Distributed dividend > Assumed dividend	Option seller	Option buyer
Distributed dividend < Assumed dividend	Option buyer	Option seller

1.4.12 Adjustments to Option Terms Due to Other Corporate Actions

Corporate actions, such as a rights issue, a stock split, a bonus issue, etc., affect the price of an option if the terms of the option are not adjusted. As a result, option terms are adjusted to mitigate the effect of these corporate actions. As an example, I will go through the adjustments to be made in case of a rights issue.

Adjustment to Option Terms as a Result of a Rights Issue

In a rights issue, the price of the existing shares automatically adjusts downwards on the ex-date to take into account the discount offered to subscribe for the new shares. As a result, when a company has a rights issue an adjustment is made on the rights issue ex-date to the option strikes and notionals to appropriately reflect the impact of the rights. The adjustment is only made if PC > PS (see below for the definitions of these two variables). The adjustment factor is calculated as follows:

$$\text{Adjustment factor} = (\text{NE} \times \text{PC} + \text{NN} \times \text{PS})/[(\text{NE} + \text{NN}) \times \text{PC}]$$

where:

NE is the number of existing shares
NN is the number of new shares
PC is the cum-rights price
PS is the subscription price

Instead of using the number of existing and new shares, it is easier to use the numbers of the issue terms. For example, assume 4 per 9 rights issue. Thus, NE = 9 and NN = 4. Let us assume that PC = 31.4 and PS = 12.0. Therefore:

$$\text{Adjustment factor} = (9 \times 31.4 + 4 \times 12.0)/[(9 + 4) \times 31.4] = 0.989897$$
$$\text{Adjusted strike} = \text{Strike} \times \text{Adjustment factor}$$
$$\text{Adjusted number of options} = \text{Number of options}/\text{Adjustment factor}$$

In this adjustment formula it is assumed that the new shares are entitled to any future dividend. However, sometimes the new shares are not entitled to an already declared dividend. In this case the dividend is added to the subscription price when calculating the adjustment factor. The formula then becomes:

$$\text{Adjustment factor} = [\text{NE} \times \text{PC} + \text{NN} \times (\text{PS} + \text{DIV})]/[(\text{NE} + \text{NN}) \times \text{PC}]$$

where:

DIV is the declared dividend per share.

1.4.13 Volatility Smile

An option premium is a function, among other variables, of the implied volatility of an underlying stock associated with a certain tenor and strike. The theoretical value of the option is derived through a theoretical pricing model such as the Black–Scholes option pricing model. One of the key assumptions of the Black–Scholes model is that the stock price follows a geometric Brownian motion with constant volatility. In other words, for a certain tenor, the model assumes that the implied volatilities of all the strikes are identical. However, in real markets implied volatility is far from constant. The volatility smile is a graph of the implied volatility of an option versus its strike, for a given tenor. It typically describes a smile-shaped curve.

The shape indicates the market belief that large movements in stock price occur with a higher probability than a theoretical option pricing model would predict, making out-of-the-money options more valuable. Figure 1.27 shows the volatility smile curve for IBM options with a 3-month time to expiration, as of 4 August 2010. It can be observed that the implied volatility in this example was lowest for 110% strikes and increased as strikes were set further. In the

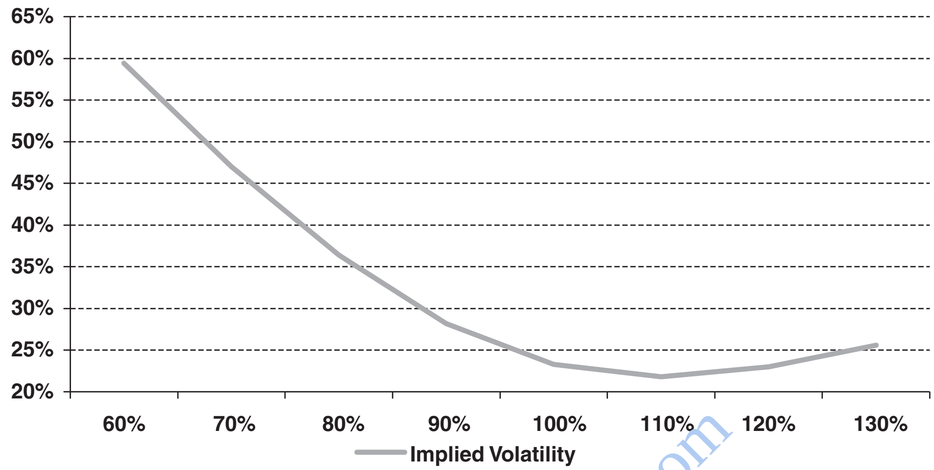

Figure 1.27 Implied volatility smile curve for 3-month IBM options, as of 4 August 2010.

equity markets the volatility smile curve is commonly not symmetrical; the volatility of low strikes is larger than for high strikes. A significant decline in a stock price typically triggers additional sales, therefore increasing the probability of a large drop, while a significant rise in the share price usually does not induce as much additional purchases.

Risk Reversal and the Skew

Skew measures the difference between the implied volatilities of a call and a put with the same delta. As we saw earlier, the implied volatility of a put option is usually larger than the implied volatility of a call option with the same delta. It means that out-of-the-money put options have a higher probability of finishing in-the-money compared with an out-of-the-money call with the same delta.

A risk reversal is the most common way to take a position in the implied volatility skew of a certain stock. A risk reversal is an option strategy that involves buying a call and selling a put, or selling a call and buying a put, with the same delta, on the same underlying and with the same expiration date. The most common risk-reversal transaction consists of trading a put and a call with a 25% delta each. For example, an investor that believes the 3-month 25% delta skew is too large (i.e., the implied volatility of the 25% delta put is too large relative to the implied volatility of the 25% delta call) will sell a 25% delta put and buy a 25% delta call, both with a 3-month expiry.

1.4.14 Implied Volatility Term Structure

The term structure of the implied volatility is a curve depicting the differing implied volatilities of at-the-money options on the same underlying with differing maturities. The scope of the curve indicates expected changes in the option market expectations for volatility over the short and long term. For example, if the market begins to expect an increase in implied volatility of a stock in the near term, the implied volatility of 1-month and 3-month options will rise

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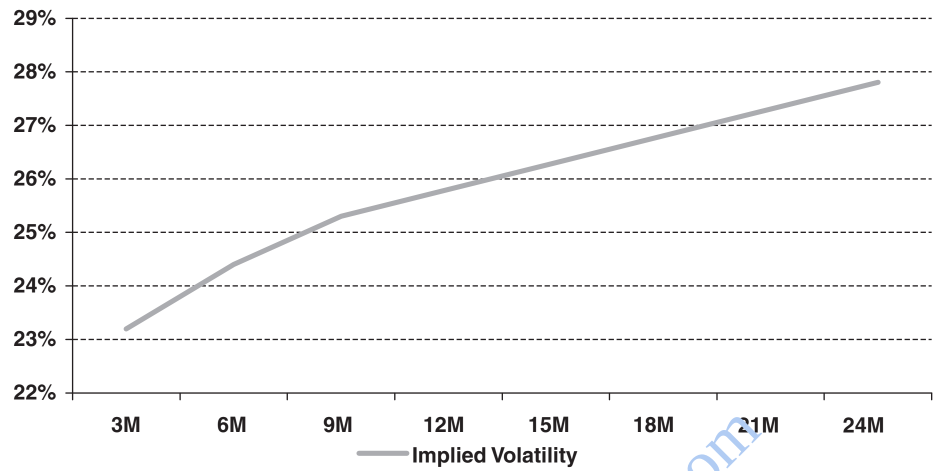

Figure 1.28 Implied volatility term structure for at-the-money options on IBM.

relative to longer-term options. If expectations are that volatility will not rise beyond a few months, the term structure might be flatter for longer-dated options. Expectations of higher near-term implied volatility will cause the term structure differential between 3-month and 1-year implied volatility to decrease. Figure 1.28 depicts IBM's term structure of volatilities as of 4 August 2010. The market expected volatilities of at-the-money options on IBM to rise in the medium term.

1.4.15 Composite and Quanto Options

First, let us assume that a USD-based investor buys a **standard call option** on Microsoft stock with a strike of K. If the stock price of Microsoft is S at expiry, the option payout at expiry would be:

$$\text{Payout} = \text{Number of options} \times \text{Max}(S - K, 0)$$

The stock price of Microsoft is denominated in USD. The option payout is also calculated in USD.

Let us assume now that a EUR-based investor buys a **composite call option** on Microsoft stock. The payout of the composite call would be:

$$\text{Payout} = \text{Number of options} \times \text{Max}(S/\text{FX}_\text{T} - K/\text{FX}_0, 0)$$

where FX_0 is the USD/EUR exchange rate prevailing at inception (to be more precise, at the time when the strike price is set) and FX_T is the USD/EUR exchange rate prevailing at expiry (to be more precise, at the time when the settlement price is calculated). The stock price of Microsoft is denominated in USD. The composite option payout is calculated in EUR. The investor is exposed to the movement of the USD/EUR exchange rate. If FX_T is lower than FX_0, the investor would benefit from a favorable FX movement.

Let us assume now that a EUR-based investor buys a **quanto call option** on Microsoft stock. The payout of the quanto call would be:

$$\text{Payout} = \text{Number of options} \times \text{Max}(S/\text{FX}_0 - K/\text{FX}_0, 0)$$

where FX_0 is the USD/EUR exchange rate prevailing at inception (to be more precise, at the time when the strike price is set). The stock price of Microsoft is denominated in USD. The quanto option payout is calculated in EUR. It can be seen that there is no FX_T in the formula. Thus, the investor is not exposed to the USD/EUR exchange rate.

1.5 DIVIDEND SWAPS

Dividend swaps are OTC derivatives that allow investors to purchase or sell the dividends paid over a specified period by a stock, a basket of stocks or an index or a combination thereof.

1.5.1 Dividend Swaps

For a given underlying, a dividend swap is the exchange, at a given maturity or periodically, of a fixed and a floating payment (see Figure 1.29). The long dividend swap counterparty will pay a fixed payment, representing the current estimated market value of dividends, and receive a floating payment, representing the distributed dividends over the period considered.

At the end of each period the fixed and floating payments are netted, resulting in the following payoff to be settled between the two parties:

$$\text{Settlement amount} = \text{Floating amount} - \text{Fixed amount}$$

The floating amount is the sum of the dividends actually paid by the underlying over the period considered.

The fixed amount is an amount set forth at the inception of the contract. It represents the expected dividends over the period considered.

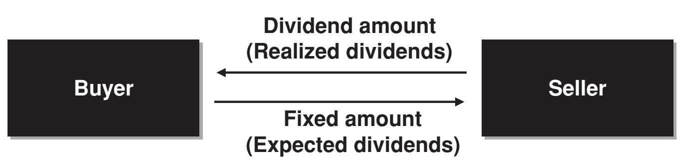

Figure 1.29 Dividend swap overview.

1.5.2 Applications of Dividend Swaps

Dividend swaps allow an investor to hedge positions involving uncertainty of dividend receipts/payments and to express views on the future levels of dividend payments.

Hedging

Investment banks with a large activity in equity derivative products to retail and institutional investors experience large exposures to dividends. The exposure stems from the short

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positions in the calls embedded in capital guaranteed equity-linked products and the long positions in the puts embedded in reverse convertibles. Together, those positions create a short forward position that has to be hedged by buying the underlying stocks which pay an unknown amount of dividends into the future. To hedge their exposure, investment banks sell dividend swaps.

An investor may want to lock in the dividends to be received from an investment in a specific stock. This is commonly the case of investors who are financing the investment. Dividends may be used to meet the debt interest payments. A decline in dividends may jeopardize an investor's capacity to meet his/her debt commitments. To hedge this exposure the investor sells a dividend swap.

Convertible bond buyers are also natural users of dividend swaps. Often convertibles are long term with little or no dividend protection. Convertible bond investors are generally short dividend risk, that is, higher than expected dividends on the underlying stock may reduce the convertible bond value. The dividend sensitivity is relatively more pronounced for convertibles with high delta and long time to maturity. This is because to delta-hedge the embedded call option a convertible bond investor needs to sell a number of shares of the underlying stock such that the overall delta remains zero. Commonly, the investor needs to borrow the shares to be sold, and therefore pay the lender an amount equivalent to the dividends distributed to the borrowed shares. An increase in the dividend means that the investor is required to pay a higher than anticipated amount of dividends to the stock lender. Therefore, convertible bond investors are natural buyers of dividend swaps.

Profiting from Directional Views

An investor may want to profit from a specific view on dividends paid by a stock or an index without holding the underlying. For example, an investor can buy dividends outright if he/she feels current implied dividend levels for a particular period are too low. Dividends are arguably the most tangible measure of the economic fundamentals of companies. Companies signal their confidence about the future by raising their dividends, while companies in difficulties are forced to cut their dividends. Unlike stocks, dividends are correlated more to fundamentals because dividends are less exposed to market risk.

Profiting from Relative Value Views

An investor may want to profit from the relative value between one stream of dividend payments and another. For example, an investor can play a steepening of the dividend term structure by buying a longer dated dividend swap and selling a shorter dated one.

Diversification

An investor may want diversification in his/her bond–equity portfolio and simultaneously improve its risk–return profile. Dividends constitute an investment that is different from investing in bonds or directly in equities. Changes in dividend growth are more gradual than stock market movements. They take longer to rise/fall in bullish/bearish markets. As a result, dividends tend to exhibit lower volatility than cash equities.

1.5.3 Risks

The potential loss for the counterparty being long the dividend swap is limited to the fixed amount paid and would occur when the underlying pays no dividend. The potential loss for the counterparty being short the dividend swap is in theory unlimited.

Because dividend swaps are OTC instruments, a default by a counterparty to the dividend swap may originate a loss to the other party.

1.5.4 Main Dates in a Dividend Distribution

When referring to dividends, there are several dates that we need to be familiar with.

The **declaration date** is the date the company's board of directors approve the size of the dividend to be proposed for approval at the next AGM (i.e., a shareholders' annual general meeting). Usually the shareholders of the company approve this recommended dividend at the AGM. Then, on a later date, the company's board of directors approves the record and payment dates.

The **record date** is the date on which an investor has to be registered as a shareholder to be granted the dividend.

The definition of the **ex-dividend date** depends on the country where the stock exchange is located, being referred to either the trade date or the settlement date:

- For US and UK stocks, for example, the ex-dividend date is the first date on which the stock will be traded without entitlement to receive the dividends. Therefore, the ex-dividend date is the day that is two exchange business days immediately prior to the record date. At the market open on the ex-dividend date, the stock trades "excluding the dividend" or "ex-dividend". Before the ex-dividend date the stock is said to be "cum-dividend" (i.e., it has entitlement to the dividend).
- For EUR denominated stocks, for example, the ex-dividend date is the first date the stock settles without entitlement to receive the dividends. Therefore, the ex-dividend date is the exchange business day immediately following the record date.

The **payment date** is the date on which the dividend is received by the shareholder.

As an example, let us assume that IBM announced a cash dividend on 18 May 20X1. The record date was 8 June 20X1. The ex-dividend date was then 6 June 20X1. In the US stock market it takes three exchange business days for a trade to settle (i.e., when the shares are delivered to the stock buyer and the proceeds are received by the stock seller). Any IBM purchasing trades settling on or before 8 June 20X1 (the record date) will be entitled to receive the dividend. Therefore, an investor buying IBM shares prior to 6 June 20X1 will be entitled to receive the dividend. For example, an investor entering into a stock purchase on 5 June 20X1 will have the trade settled on 8 June 20X1, thus being entitled to receive the dividend. A purchase trade entered into on 6 June 20X1, the ex-dividend date, would be settled on 9 June 20X1 (after the record date) and, therefore, will not be entitled to receive the dividend.

1.5.5 Case Study: Single-stock Dividend Swap

Let's assume that on 31 December 20X0 Zurich Bank thinks that XYZ Corp.'s implied dividend levels for 20X1 are too conservative. XYZ already cut its dividend in 20X0, and the market is still very negative regarding XYZ's upcoming dividends. Zurich Bank thinks that XYZ's fundamentals are strong and it will be able to distribute higher cash dividends in the near future. As a result, Zurich Bank buys a dividend swap from Gigabank with the following terms:

Dividend Swap on a Single Stock – Main Terms	
Fixed amount payer (i.e., the buyer)	Zurich Bank
Dividend amount payer (i.e., the seller)	Gigabank
Trade date	31-December-20X0
Effective date	31-December-20X0
Termination date	31-December-20X2
Underlying stock	XYZ Corp. common stock (ordinary shares)
Exchange	Euronext
Related exchange	Eurex
Number of shares	10 million
Fixed strike	EUR 0.85, per stock
Fixed amount per dividend period	EUR 8.5 million (= Number of shares × Fixed strike)
Dividend amount per dividend period	Number of shares × $\sum_{t=1}^{T}(d_t)$
Declared cash dividend percentage	100%
Declared cash equivalent dividend percentage	100%
Dividend periods	From 31-Dec-20X0 to 31-Dec-20X1, and from 1-Jan-20X2 to 31-Dec-20X2
Settlement amount payment date	One currency business day following the end date of the relevant dividend period
Settlement currency	EUR
Calculation agent	Gigabank

where:

Underlying is the stock or index whose dividends are being bought or sold.

Fixed strike is an amount set forth at the inception of the transaction, which can be interpreted as the implied dividend for one share during a dividend period.

Fixed amount is the product of (i) the number of shares and (ii) the fixed strike.

Dividend period(s) or valuation period(s) are the period(s) within which paid dividends will qualify. Commonly, a dividend period comprises a calendar year.

Settlement amount payment date(s) is the date(s) on which the payoff is settled. Settlement typically occurs one business day after each period ends. At each settlement date, amounts are netted and the settlement amount is computed. If the settlement amount is positive, the seller pays the buyer the settlement amount. If the settlement amount is negative, the buyer pays the seller the absolute value of the settlement amount:

$$\text{Settlement amount} = \text{Dividend amount} - \text{Fixed amount}$$

Dividend amount or distributed amount is the sum of all the dividends paid by the underlying during the dividend period. The dividend amount is calculated as the product of (i) the number of shares and (ii) the sum of all the d_t dividends distributed to one share during the dividend period:

$$\text{Dividend amount} = \text{Number of shares} \times \sum_{t=1}^{T} (d_t)$$

t means each weekday in the relevant dividend period, and T is the total number of weekdays in the relevant dividend period.

Number of shares is the underlying quantity upon which payment obligations are computed. It can be viewed as the amount to be paid for a one-point difference between the fixed amount and the dividend paid per share.

Qualifying dividend d_t. In respect of the underlying stock and each day t in the relevant dividend period: (i) if an ex-dividend date falls on such day, an amount equal to the "relevant dividend"; or (ii) otherwise zero. Distributed dividends are generally defined as a percentage (100% in our case) of the ordinary cash or stock dividends declared in the currency of the company's announcement. Dividends are considered before any applicable withholding tax and disregarding any tax credit. Stock dividends are included at the equivalent cash amount. Note that dividends are not reinvested in the money markets. Dividends are neither compounded nor capitalized.

Relevant dividend. In respect of the underlying stock and each day t in a dividend period: (a) the "declared cash dividend"; and/or (b) the "declared cash equivalent dividend", excluding any dividends in relation to which the related primary exchange adjusts the derivatives contracts that include the underlying.

Declared cash dividend. In respect of a relevant dividend, an amount per share as declared by the issuer where the ex-dividend date falls on such day t, before the withholding or deduction of taxes at source by or on behalf of any applicable authority having power to tax in respect of such dividend, and shall exclude: (a) any imputation or other credits, refunds or deductions granted by an applicable authority; and (b) any taxes, credits, refunds or benefits imposed, withheld, assessed or levied on the credits referred to in (a) above.

Declared cash equivalent dividend. In respect of a relevant dividend, an amount per share being the cash value of any stock dividend (whether or not such stock dividend comprises of shares that are not the common shares of the issuer) declared by the issuer where the ex-dividend date falls on such day t (or, if no cash value is declared by the relevant issuer, the cash value of such stock dividend as determined by the calculation agent, calculated by reference to the opening of such common shares on the ex-dividend date applicable to that stock dividend).

If holders of record may elect between receiving a declared cash dividend or a declared cash equivalent dividend, the dividend is deemed to be a declared cash dividend.

Where any relevant dividend is declared in a currency other than the settlement currency, then the calculation agent shall convert such relevant dividend into the settlement currency at the rate declared by the issuer, where any such rate is available or, if no such rate is available, at a rate determined by the calculation agent.

Ex-dividend date. In respect of a relevant dividend, the date that the share is scheduled to trade ex-dividend on the primary exchange or quotation system for such share.

Termination date is the date on which the contract ends.

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In our example, the first dividend period started on 31 December 20X0 and ended on 31 December 20X1. Let us assume that XYZ distributed the following dividends around those dates:

Dividend number	Ex-dividend date	Terms	XYZ opening price on ex-dividend date	Derivatives adjustment on related exchange?
1	30-Dec-X0	EUR 0.30 cash	NA	No
2	1-Apr-X1	EUR 0.55 cash	NA	No
3	1-Jul-X1	1 new share for each 50 shares	EUR 20.00	No
4	1-Oct-X1	1 new share for each 30 shares	EUR 21.00	Yes
5	1-Nov-X1	EUR 1.20 cash	NA	Yes

In order to check if a dividend constitutes a "qualifying dividend", we have to check (i) if the ex-dividend date falls within the dividend period and (ii) if the terms of the derivatives (usually futures and options) on the underlying stock that trade on the related primary exchange are not adjusted.

Companies sometimes pay special or extraordinary dividends, which are usually excluded from the calculation of the qualifying dividends. In order to avoid arguing whether a specific dividend constitutes a qualifying dividend, contracts follow the related exchange rules. Dividends that cause an adjustment to derivatives on the same underlying (the designated contract) by the related primary exchange are not included. Therefore:

- Dividend 1 is not a qualifying dividend because its ex-dividend date is prior to the considered dividend period.
- Dividend 2 is a qualifying dividend because its ex-dividend date is within the considered dividend period and it does not cause an adjustment to the derivatives trading in the related primary exchange on XYZ Corp. Therefore, the "declared cash dividend" is EUR 0.55 and it will be taken into account when computing the dividend amount.
- Dividend 3 is a qualifying dividend because its ex-dividend date is within the considered dividend period and it does not cause an adjustment to the derivatives trading in the related primary exchange on XYZ Corp. It is a scrip dividend. For each existing share, the shareholder of XYZ received 1/50 shares. It is assumed that this 1/50 share is sold at the opening of the market on the ex-dividend date. XYZ shares were trading at that moment at EUR 20.00. Therefore, the "declared cash equivalent dividend" amount is EUR 0.40 (= 20/50).
- Dividend 4 is not a qualifying dividend because it causes an adjustment to the derivatives trading in the related primary exchange on XYZ Corp.
- Dividend 5 is not a qualifying dividend because it causes an adjustment to the derivatives trading in the related primary exchange on XYZ Corp.

As a consequence, the sum of the qualifying dividends is EUR 0.95 (= 0.55 + 0.40). The dividend amount for the period would be calculated as:

$$\text{Dividend amount} = \text{Number of shares} \times \text{Sum of qualifying dividends}$$
$$\text{Dividend amount} = 10 \text{ million} \times 0.95 = \text{EUR } 9.5 \text{ million}$$

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The fixed amount was EUR 8.5 million. Thus, the settlement amount was EUR 1 million (= 9.5 million – 8.5 million). Because the settlement amount was positive, the fixed amount payer – Zurich Bank – received from the dividend amount payer – Gigabank – the EUR 1 million settlement amount on the settlement amount payment date – 1 January 20X2. Remember that there was a second dividend period that I will not cover as the mechanics to calculate the settlement amount are identical.

1.5.6 Case Study: Index Dividend Swap

Index dividend swaps give exposure to the dividends distributed by the stocks that are members of the index during a specific period. The mechanics of an index dividend swap are very similar to a single stock dividend swap. The only difference is a notably more complex computation of the actual dividends. As an example, let us assume that on 31 December 20X0 Zurich Bank thinks that EuroStoxx 50 implied dividend levels for 20X1 and 20X2 are too low. In 20X0 there has been a deep recession that has obliged auto and construction companies to skip dividends. Thus, banks' trading desks have been forced to sell their losing large long dividend positions. Zurich Bank thinks that as a result of the widespread unwinding of trades, the market dividend levels for the EuroStoxx 50 are discounting an unprecedented dividend cut for 20X1 and 20X2. As a result, Zurich Bank buys a two-year dividend swap on the EuroStoxx 50 from Gigabank with the following terms:

Dividend Swap on an Index – Main Terms	
Fixed amount payer (i.e., the buyer)	Zurich Bank
Dividend amount payer (i.e., the seller)	Gigabank
Trade date	31-December-20X0
Effective date	31-December-20X0
Termination date	31-December-20X2
Underlying index	EuroStoxx 50
Exchange	Euronext
Related exchange	Eurex
Number of baskets	100,000 (for avoidance of doubt, the number of baskets is in EUR and 10,000 contracts of EUR 10 tick value each)
Fixed strike	120 index points, per basket
Fixed amount per dividend period	EUR 12 million (= Number of baskets × Fixed strike)
Dividend amount per dividend period	Number of baskets × $\sum_{t=1}^{T}\sum_{i=1}^{N}\left(\frac{n_{i,t}\times d_{i,t}}{D_t}\right)$
Declared cash dividend percentage	100%
Declared cash equivalent dividend percentage	100%
Dividend periods	From 31-Dec-20X0 to 31-Dec-20X1, and from 1-Jan-20X2 to 31-Dec-20X2
Settlement amount payment date	One currency business day following the end date of the relevant dividend period
Settlement currency	EUR
Calculation agent	Gigabank

The settlement amount is calculated as:

$$\text{Settlement amount} = \text{Dividend amount} - \text{Fixed amount}$$

The dividend amount is calculated as follows:

1. The total dividends paid by each company member of the index during the period considered are computed.
2. All the dividends calculated in (1) are summed up to arrive at a figure for the total dividends paid out on the index.
3. The amount calculated in (2) is divided by the index divisor to arrive at the dividends paid in index points.
4. The figure calculated in (3) is multiplied by the number of baskets.

More precisely, the dividend amount is calculated as follows:

$$\text{Dividend amount} = \text{Number of baskets} \times \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\frac{n_{i,t} \times d_{i,t}}{D_t}\right)$$

where:

Number of baskets is the predetermined number of index points. It can be viewed as the amount to be paid for a one-point difference between the fixed amount and the dividend paid, weighted according the index.

t means each weekday in the relevant dividend period.

i means, in respect of each day t, each share that is comprised in the index on such day.

$n_{i,t}$ means, in respect of each share i and day t, the number of free-floating shares relating to such share comprised in the index, as calculated and published by the index sponsor on such day t.

The qualifying dividend $d_{i,t}$ means, in respect of each share i and day t: (i) if an ex-dividend date falls on such day, an amount equal to the relevant dividend; or (ii) otherwise zero.

D_i means, in respect of each day t, the official index divisor, as calculated and published by the index sponsor in respect of such share i and such day t.

Official index divisor is the value, calculated by the index sponsor, necessary to ensure that the numerical value of the index remains unchanged after a change in the composition of the index. The value of the index after any change in the composition is divided by the official index divisor to ensure that the value of the index returns to its normalized value.

Relevant dividend means, in respect of each share i and day t in a dividend period: (a) the "declared cash dividend percentage" specified in the transaction terms of any declared cash dividend; and/or (b) the "declared cash equivalent percentage" specified in the relevant transaction terms of any declared cash equivalent dividend, excluding any dividends in relation to which the index sponsor makes an adjustment to the index. Where the index sponsor has adjusted the index for part of a dividend, this relevant dividend provision applies only to the unadjusted part.

Declared cash dividend means, in respect of a relevant dividend of share i, an amount per share i as declared by the issuer of such share i where the ex-dividend date falls on such day t, before the withholding or deduction of taxes at source by or on behalf of any applicable authority having power to tax in respect of such dividend, and shall exclude: (a) any imputation or other credits, refunds or deductions granted by an applicable authority; and (b) any taxes, credits, refunds or benefits imposed, withheld, assessed or levied on the credits referred to in (a) above.

Declared cash equivalent dividend means, in respect of a relevant dividend of share *i*, an amount per share *i* being the cash value of any stock dividend (whether or not such stock dividend comprises of shares that are not the common shares of the issuer) declared by the issuer of such share *i* where the ex-dividend date falls on such day *t* (or, if no cash value is declared by the relevant issuer, the cash value of such stock dividend as determined by the calculation agent, calculated by reference to the opening of such common shares on the ex-dividend date applicable to that stock dividend).

If holders of record of share *i* may elect between receiving a declared cash dividend or a declared cash equivalent dividend, the dividend is deemed to be a declared cash dividend.

Where any relevant dividend is declared in a currency other than the "settlement currency", then the calculation agent shall convert such relevant dividend into the settlement currency at the rate declared by the issuer where any such rate is available or, if no such rate is available, at a rate determined by the calculation agent.

Ex-dividend date means, in respect of a relevant dividend, the date that share *i* is scheduled to trade ex-dividend on the primary exchange or quotation system for such share *i*.

Termination date is the date on which the contract ends.

Let us assume that the actual dividends paid by the stocks that are members of the EuroStoxx 50 index from 31 December 20X0 to 31 December 20X1 (the first dividend period) were 180 index points:

The dividend amount would be 18 million (= 100,000 × 180)
The fixed amount was 12 million (= 100,000 × 120)
The settlement amount would be EUR 6 million (= 18 million – 12 million)

As a consequence, the dividend swap buyer (Zurich Bank) received EUR 6 million one currency business day after 31 December 20X1.

1.5.7 Pricing Implied Dividends

The implied dividend of a stock can be calculated by using the put–call parity formula (as explained earlier in this chapter):

$$\text{Stock} + \text{Put} = \text{PV(Strike)} + \text{Call} + \text{PV(Dividends)} + \text{PV(Repo)}$$

where "PV" is the present value.

Solving for the dividends:

$$\text{PV(Dividends)} = \text{Stock} + \text{Put} - \text{Call} - \text{PV(Strike)} - \text{PV(Repo)}$$

1.6 VARIANCE SWAPS AND VOLATILITY SWAPS

Variance and volatility swaps give "pure" exposure to volatility. Volatility can also be traded using options, for example by trading a straddle, but an investment in a variance/volatility

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swap can be more efficient as it requires little re-hedging, unlike a gamma hedging option position. Another benefit of variance and volatility swaps is their zero upfront premium.

Variance swaps are OTC derivatives that allow investors to purchase or sell the realized variance, defined as the square of future realized volatility, over a specified period by a stock or an index or a combination thereof.

Volatility swaps are OTC derivatives that allow investors to purchase or sell the realized volatility over a specified period by a stock or an index or a combination thereof.

1.6.1 Variance Swaps Product Description

The holder of a EUR denominated variance swap at expiration receives a EUR "variance amount" for every point by which the stock realized variance has exceeded an expected predetermined variance amount. No premium is paid upfront to enter into the variance swap. For a given underlying, a variance swap is the exchange, at a given maturity or periodically, of a fixed and a floating amount (see Figure 1.30). At maturity, or at the end of each period, the fixed and the floating payments are netted between the two parties:

$$\text{Settlement amount} = \text{Floating amount} - \text{Fixed amount}$$

The floating amount represents the underling realized variance over the period:

$$\text{Floating amount} = \text{Variance amount} \times \text{Realized volatility}^2$$

The fixed amount represents the underlying expected variance over the period:

$$\text{Fixed amount} = \text{Variance amount} \times \text{Volatility strike}^2$$

As a result, the settlement amount can be computed as:

$$\text{Settlement amount} = \text{Variance amount} \times (\text{Realized volatility}^2 - \text{Volatility strike}^2)$$

If the settlement amount, also called the **equity amount**, is positive the seller pays the buyer the settlement amount. Conversely, if the settlement amount is negative the buyer pays the seller the absolute value of the settlement amount.

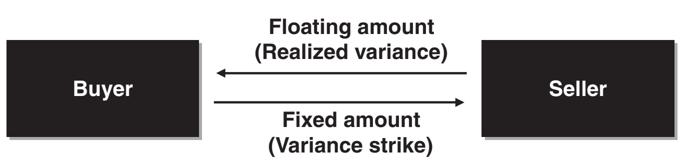

Figure 1.30 Variance swap flows.

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Definitions

Realized volatility² is the realized stock variance (quoted in annual terms) over the life of the contract, expressed in percentage.

Volatility strike², also called **variance strike price**, is the delivery price for variance that makes the swap have zero value.

Variance amount, also called variance units, is the notional amount of the swap per unit of currency per annualized volatility point squared.

Termination date is the date on which the settlement amount is paid. It is usually three scheduled trading days after the observation end date.

As an example, let us assume that ABC believes that the market is estimating a too low level of future variance of XYZ's stock price over the next year. As a result, ABC enters into the following variance swap with Gigabank:

Variance Swap – Main Terms	
Buyer	ABC
Seller	Gigabank
Trade date	30-December-20X0
Termination date	5-January-20X2
Underlying	XYZ Corp
Observation start date	2-January-20X2
Observation end date	2-January-20X3
Variance amount	5,000
Volatility strike	25
Settlement date	Three exchange business days after the observation end date

In our example, if the realized volatility over the period was 30%, the buyer – ABC – would receive from the seller – Gigabank – a settlement amount equal to EUR 1,375,000, calculated as follows:

$$\begin{aligned}\text{Floating amount} &= \text{Variance amount} \times \text{Realized volatility}^2\\ &= 5{,}000 \times 30^2 = \text{EUR } 4{,}500{,}000\\ \text{Fixed amount} &= \text{Variance amount} \times \text{Volatility strike}^2\\ &= 5{,}000 \times 25^2 = \text{EUR } 3{,}125{,}000\\ \text{Settlement amount} &= \text{EUR } 4{,}500{,}000 - 3{,}125{,}000 = 1{,}375{,}000\end{aligned}$$

Conversely, if the realized volatility over the period was 15%, the buyer – ABC – would pay to the seller – Gigabank – the absolute value of the settlement amount (i.e., EUR 2,000,000), calculated as follows:

$$\begin{aligned}\text{Floating amount} &= \text{Variance amount} \times \text{Realized volatility}^2\\ &= 5{,}000 \times 152 = \text{EUR } 1{,}125{,}000\\ \text{Fixed amount} &= \text{Variance amount} \times \text{Volatility strike}^2\\ &= 5{,}000 \times 252 = \text{EUR } 3{,}125{,}000\\ \text{Settlement amount} &= \text{EUR } 1{,}125{,}000 - 3{,}125{,}000 = -2{,}000{,}000\end{aligned}$$

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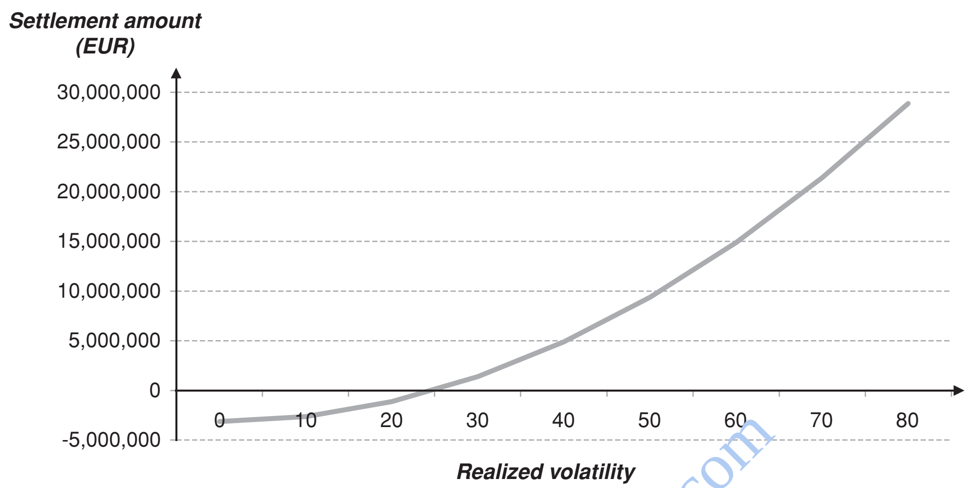

Figure 1.31 Settlement amount as a function of the realized volatility.

Figure 1.31 shows the settlement amount as a function of the realized volatility. The graph highlights the convexity profile of the settlement amount, providing an advantage to the buyer. In other words, for a large movement of the realized volatility, the buyer will benefit more from being right than he/she will lose from being wrong.

1.6.2 Calculation of the Realized Volatility and the Realized Variance

The formula to calculate the realized volatility or variance is defined at the beginning of the contract. The realized volatility σ_R is the annualized standard deviation of a stock's return during a period, expressed in percentage. A common formula is the following:

$$\text{Realized volatility} = \sigma_R = 100 \times \sqrt{\frac{252 \times \sum_{i=1}^{N} \left(Ln \frac{P_i}{P_{i-1}} \right)^2}{\text{Expected } N}}$$

This formula assumes that the stock price follows a log-normal distribution, one of the assumptions of the Black–Scholes model. It is important to note that stock prices in the formula are not adjusted for dividends. A stock with a high dividend yield may seem to be more volatile than a low dividend yielding stock.

Definitions

Ln is the natural logarithm.

P_i and P_{i-1} are the official levels of the underlying on respectively the *i*th and *i*–1th observation days. In most cases the official level is the daily closing price of the underlying.

N is the actual number of realized trading days for the period from, but excluding the observation start date to, and including, the observation end date.

252 is the **annualization factor**. Usually, it is assumed to be 252 trading days per year.

Expected N is the number of days that, as the trade date, are expected to be scheduled trading days for the period from, but excluding the observation start date to, and including, the observation end date. In other words, "expected N" is the number of pre-agreed observation days.

Observation day is each trading day during the observation period.

Observation period is the period from, but excluding, the observation start date to, but excluding, the observation end date.

The **realized variance** is calculated as:

$$\text{Realized variance} = (\text{Realized volatility})^2$$

1.6.3 Volatility Swaps Product Description

The holder of a EUR denominated volatility swap at expiration receives the EUR "volatility amount" for every point by which the stock realized volatility has exceeded an expected predetermined volatility amount. No premium is paid upfront to enter into the volatility swap. For a given underlying, a volatility swap is the exchange, at a given maturity or periodically, of a fixed and a floating amount (see Figure 1.32). At maturity, or at the end of each period, the fixed and the floating payments are netted between the two parties:

$$\text{Settlement amount} = \text{Floating amount} - \text{Fixed amount}$$

The floating amount represents the underlying realized volatility over the period:

$$\text{Floating amount} = \text{Volatility amount} \times \text{Realized volatility}$$

The fixed amount represents the underlying expected volatility over the period:

$$\text{Fixed amount} = \text{Volatility amount} \times \text{Volatility strike}$$

As a result, the settlement amount can be computed as:

$$\text{Settlement amount} = \text{Volatility amount} \times (\text{Realized volatility} - \text{Volatility strike})$$

If the settlement amount, also called the **equity amount**, is positive the seller pays the buyer the settlement amount. Conversely, if the settlement amount is negative the buyer pays the seller the absolute value of the settlement amount.

Volatility amount, also called **volatility units**, is the notional amount of the swap per unit of currency per annualized volatility.

For definitions of the other terms, please refer to the variance swap product description.

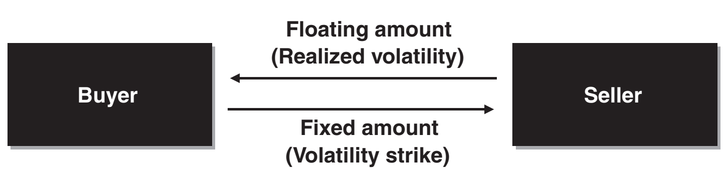

Figure 1.32 Volatility swap flows.

1.6.4 Volatility Swaps vs. Variance Swaps

Although options market participants talk about volatility, it is variance, or volatility squared, that has more fundamental theoretical significance. This is so because the correct way to value a swap is to value the portfolio that replicates it and the swap that can be replicated most reliably is a variance swap. A variance swap is a forward contract on annualized variance, the square of the realized volatility.

The payoff of a variance swap is convex in volatility, as illustrated in Figure 1.31. This means that an investor who is long a variance swap (i.e., receiving realized variance) will benefit from a larger amount if he/she is right than if he/she is wrong, for the same movement in volatility. This bias has a cost reflected in a slightly higher strike than the "fair" volatility. Thus, volatility swaps are a cheaper investment as a buyer, as Volatility $\text{swap}_{\text{price}} <$ Variance $\text{swap}_{\text{price}}$.

Similarly, volatility swaps have a better risk profile as a seller, as Volatility $\text{swap}_{\text{convexity}}$ $<$ Variance $\text{swap}_{\text{convexity}}$.

An interesting property of variance swaps is their additivity. For example, the sum of a 3-month variance and a 9-month variance in 3 months is equal to a 12-month variance.

	Variance swaps	Volatility swaps
Pure volatility exposure	√	√
Positive convexity	√	
Implied/realized additivity	√	

1.6.5 Applications of Variance and Volatility Swaps

Variance and volatility swaps allow an investor to hedge positions involving uncertainty of volatility and to express views on future levels of volatility. A stock volatility is the simplest measure of its riskiness. Although volatility can be traded with options, volatility swaps provide a much more direct method.

Hedging against Volatility Movements

A portfolio of equity derivatives hedged with positions in the underlying stocks leaves exposure to realized volatility, which can be hedged by taking a position in variance or volatility swaps.

Also, risk arbitrageurs often take positions which assume that the spread between the stock prices of two companies planning to merge will narrow. If overall market volatility increases, the merger may become less likely and the spread may widen. By buying a volatility swap, an arbitrageur can put in place a proxy hedge.

Directional Views on Volatility

Variance and volatility swaps provide pure exposure to volatility. Volatility has several characteristics that make trading attractive. It is likely to grow when uncertainty and risk increase. As with interest rates, volatilities appear to revert to the mean; high volatilities will eventually decrease, low ones will likely rise. If an investor thinks current volatility is low, for the right price he/she might want to take a position that profits if volatility increases. For example, if an investor foresees a rapid decline in financial turmoil after a specific crisis, a short position in volatility might be appropriate.

Trading the Spread between Realized and Implied Volatility

Variance/volatility swaps allow an investor to capture the premium between implied and realized volatility. For example, let us assume that an investor buys a variance/volatility swap in a period in which stock prices are experiencing large moves while the implied volatility is much lower. The investor will benefit from a favorable carry associated with being long volatility.

Relative Value Views on Future Volatility

An investor who wants to take a view on the future levels of stock or index volatility can go long or short a future realized volatility. Forward trades are interesting because the forward volatility term structure tends to flatten for longer forward start dates. Because variance is additive, an investor can obtain a perfect exposure to forward implied volatility. For example, an investor can be long future realized volatility by buying a longer dated variance swap and selling a shorter dated one.

Trading Correlation (Dispersion Trades)

An investor may be looking to buy correlation by taking a long position on an index variance/ volatility and a short position in the variance/volatility of the index components. Commonly, only the most liquid stocks are chosen among the index components and each variance/ volatility swap notional is adjusted to match the same volatility sensitivity as the index.

Diversification

An investor may want diversification in his/her bond–equity portfolio and simultaneously improve its risk–return profile. Equity volatility constitutes an investment that is different from investing in bonds or directly in equities. Volatility is often negatively correlated with a stock or an index level, and tends to stay high after large downward moves in the market. Investing in a variance/volatility swap can bring diversification to a portfolio.