## INDEX



## A

abaci schools, 26
actuarial science, 141-142
actuaries, $34,49,61,141,177$
adverse selection, 67
age, and human capital, 111
aging, and mortalitv, 49
AIG, 124, 12.6
Al-Kwarieini, Muhammed, 19
Alexandrov, Pavel Sergeyich, 173
Allen, Robert, 96
Alliance Assurance Company, 52
American College, 132
American Economics Association, 95, 123
American Society of Actuaries (SoA), 141
annuities, 4, 167
see also Halley's equation; pensions
and adverse selection, 67
annuity puzzle, 146-147
for de-accumulation of wealth, 146
French treasury, 54
and group pricing, 68
proper pricing and valuation, 54
Solomon Huebner, 144-147
annuity factor tables, 66
annuity puzzle, 146-147
Armstrong, William, 126
Aryabhatiya, 175
asset allocation equation. See
Samuelson's equation
Astronomical Society, 52
Austen, Jane, 67, 145

## B

baby boomers, 2
Bailey, Francis, 141
Bank of England, 72-73, 72f
Becker, Gary, 110

## INDEX

Bengen, William, 81-82
Bernstein, Peter, 177
Bismarck, Otto von, 6
Black, Fischer, 177
Black Thursday, 99
Bodie, Zvi, 107, 116, 119
bond market, 65-66
Breslau, Poland, 55, 69
bubble tendencies, 98
buy-term-and-invest-the-difference
(BTID) philosophy, 143-144

## C

cafeteria Keynesian, 122
calculus, 159
capital asset pricing model (CAPM) equation, 177
Capital Ideas: The Improbable Origins of Modern Wall Street
(Bernstein), 177
capital market, 85
causes of death, 49
centenarians, 48, 49-50
see also Gompertz law of mortality
Charles II, King of England, 55-56
Chartered Life Underwriter (CLU), 132
Chicago school, 121-122
classical economics, 86
Clemens, Samuel, 76
Clinton, Bill, 123
common logarithms, 11, 180
compound interest calculations, 20
compounding, 20-21, 182-183
conclusion, 175-178
consumption rate, $84-85,90 f, 91 f$ see also spending rate
consumption smoothing, 82, 94
continuous state-space Markov process, 171
continuous time Markov process, 171
Cramer, Harald, 159
credit, 175-177
credit card interest, 85-86

## D

de-accumulation planning, 2
de Witt, Jan, 176
death, and probability of dying, 37
death benefit, 134, 140
see also life insurance; life insurance equation
death rates, 160-161
Defined Benefit (DB) plan, 56-58
Defined Contribution (DC) plan, 32, 57-58
demographers, 49, 54
derivatives, 160, 162
"Determining Withdraval Rates Using Historcal Data" (Bengen), 82
differentialeguation, 160-161, $1 \div-168,170,172$
see riso Kolmogorov's equation
liscount rate, 84, 85-86, 88-89
di. Sutilities of loss, 102
divergent Fourier series, 152
diversified portfolio, 107, 149-150
Dmitrievna, Anna, 173
dollar-valued withdrawal rate, 155
Dow Jones Industrial Average (DJIA), 102, 104, 105f, 123-124

## E

$e=2.71828,180-181,182-183$
early death of a parent, 120
economic tradeoff, 15, 50-51
The Economics of Life Insurance (Huebner), 129, 141
Economics (Samuelson), 120
Edgeworth, Francis Ysidro, 95
Edmonds, T.R., 176
efficient investment frontier, 177

Einstein, Albert, 2, 32
Eisenberg, Lee, 22
equations
beauty of, 3
capital asset pricing model (CAPM) equation, 177
differential equation, 160-161, 167-168, 170, 172
Fibonacci's equation. See Fibonacci's equation
of finance, 177-178
Fisher's equation. See Fisher's equation
Gompertz's equation. See Gompertz law of mortality
Halley's equation. See Halley's equation
Huebner's equation. See life insurance equation
Kolmogorov's equation. See Kolmogorov's equation
life insurance equation. See life insurance equation
missing equations, 177-178
option pricing equation, 123,177
Samuelson's equation. See Samuelson's equation
stochastic differentiol equation (SDE), 172
equity. See stocks
eugenics morernent, 98
Euler, Leorard, 23
exchange-traded fund (ETF), 115
expected return, 112, 161

## F

Fibonacci, Leonardo, 3, 156-157
see also Fibonacci's equation
contributions to commercial mathematics, 17-21
contributions to financial economics, 26

Fibonacci series (Fibonacci
numbers), 17, 27-29
Hindu-Arabic number system, 9, 25, 26
interest discounting, 20
life story, 7-9, 24-26
and lifetime horizon, 33
On Problems of Travellers and Also Similar Problems, 18-20
retirement, 29
On a Soldier Receiving Three
Hundred Bezants for His
Fief, 20
On a Ton of Pisan Cheese, 20
On Two Men Whe Had a Company
in Constaitinople, 20-21
Fibonacci series (Fibonacci numbers), 17.27-29

Fibonacis equation
seéa! so Fibonacci, Leonardo
1tanipulation of, 21-23
present value, 20, 23-24
spending rate, 9-17
financial capital, 103, 109, 110, 115
financial crisis of 2008, 177-178
financial illiteracy, 3
financial risk, 5
Fisher, Irving, 5, 13-14, 50, 87, 120
see also Fisher's equation
on banking, 94
contributions to retirement economics, 78, 79
dissertation, 94
disutilities of loss, 102
on eugenics, 98
and financial irrationality, 95
health, 96
index card system, 95
inflation index, 80
as inventor, 95-96
life story, 78-79, 93-98

## INDEX

lifecycle consumption smoothing, 82, 94
"Narrow Banking", 94
personal tragedies, 93
and psychology, 95
rational consumers, 78
retirement, 99-100
vs. Solomon Huebner, 147-150
on spending rate, 91-92
and stock market crash, $98-100$
writings, 94, 96-98
Fisher's equation
see also Fisher, Irving
application of, 88-91
confirming the numbers, $92-93$
consumption rate, 84-85, 90f, $91 f$
explained, 84-88
longevity risk aversion, 86-87, 88-89, 88-90, 92, 93
rational adjustment of consumption, $90 f, 91 f$
subjective discount rate, $84,85-86$, 88-89, 92
variables, 84-87
in words, $87-88$
Fisher's inflation equation, 80
Fisk, Eugene Lyman, 96-97
fluxional notation for calculuc. 1
$4 \%$ rule, 81 - 82
Frederick II, Emperor of Rome, 29
French Revolution, 54
Friedman, Milton 121, 122

## G

Gambler's Ruin problems, 158-159, 172, 176
Gaunt, John, 54
gender, and mortality, 44-45
General Theory (Keynes), 122
Gibbs, Josiah Willard, 94
Goetzmann, William, 21
Golden Ratio, 28

Gompertz, Benjamin, 4, 155, 176
see also Gompertz law of mortality
on causes of death, 49
discovery, 34-38
on law of mortality, 46
life story, 31-32, 51-52
probability of dying, 37
research, 34
Gompertz law of mortality, 32
see also Gompertz, Benjamin
accuracy of equation, 47-50
at advanced ages, 48
for advanced ages, 49-50
application to more values, 41-43, $42 t$
detailed example, 39-41
different $(x)$ and $(t)$ velues, 39-41
discovery of, $37.33^{\circ}$
gender, effect ©f, 44-45
internal limits of the equation, 4 4- 17
median remaining lifetime, 41, 44-45
modal value of life, 39
modern problem with, 49
negative senescence, 50
parameters, 38-39
probability of survival, 41-43, 42t, 45-46, 49-50, 160
and retirement planning, 50-51
using, 38-39
Gorbachev, Mikhail, 153
government bonds, 65-66, 107
Great Depression, 79, 98-100, 121
group pricing, 68

## H

Haberman, Steven, 141
Halley, Edmond, 4, 34, 120, 155, 176
see also Halley's equation
academic career, 74
article by, 55
atheism, 74
calculations, 72
family life, 74
father's death, effect of, 55-56, 75-76
Halley's comet, 60, 75-76
life story, 54-55, 59-60, 73-75
mortality estimates, 72
and Newton's Principles, 60
titles, 60
on valuation of annuities, 62
on valuation of life insurance, 142
Halley, Edmond Sr., 55-56, 74
Halleyan lines, 60
Halley's comet, 60, 75-76
Halley's equation
see also Halley, Edmond
and adverse selection, 67
annuity factor tables, 66
application of, 69-73
components of, 61
demonstration of, 63-66
example, 61-62
explained, 61-63
and group pricing, 68
and market prices, 73
pension value, and age and interest, 69-73
real $v$ s. nominal apllars, 68
technical issues, 66-68
Hindu-Arabic 1umber system, 9, 25,26
The History of Fish, 73
H1N1, 130
Holland, 176
How to Live: Rules for Healthful Living Based on Modern Science (Fisher and Fisk), 96-98
Huebner, Grover, 148
Huebner, Solomon, 6, 156, 177
see also life insurance equation
American College, founding of, 132
on annuities, 144-147
annuity puzzle, 146-147
economics of insurance, 129, 141
human life value, 128-132, 145
vs. Irving Fisher, 147-150
on life insurance, 129-132
life story, 127-128, 144, 147-150
stock diversification philosophy, 149-150
Hughes, Charles E., 126
human capital, 103, 109-111, 112, 116,119
human life value, 128-132, 145
Huygens, Christiaan, 158-159, 176

## I

index card syste~1, 95
infinite serics. 27
infinity, 1617
inflafion.
rioher's inflation equation, 80
inflation-adjusted dollars, 68
inflation adjustments, 71
inflation index, 80
and retirement, 79-80
inflation index, 80
instantaneous force of mortality (IFM), 160-161, 169-170
insurance industry, 125-128
see also Huebner, Solomon; life insurance
"insurance men," 126
interest
compound interest calculations, 20
compounding, 20-21, 182-183
credit card interest, 85-86
nominal interest, 13
real interest, 13-14
interest discounting, 20
interest rates
and bond prices, 66
in England, 17th century, 72-73, 72f

## INDEX

knowing future rates, 24
major Italian business cities (12001400), $25 f$
nominal interest rate, 80
and pension value, 65-66
real interest rate, $21,80,84,85$
from safest possible investment, 112-113
subjective discount rate, $84,85-86$, 88-89, 92
Internal Revenue Service (IRS), 99
intertemporal choice, 88
irrational number, 180-181
Ito, Kiyosi, 171-172

## J

Jackson, Michael, 121
James, Prince of England, 55-56
job security, 116
Johnson, Lyndon, 122

## K

Kataev, Nikolai, 173
Kennedy, John F., 120, 122
Keynes, John Maynard, 78, 122
Keynesian economics, 122
Kolmogorov, Andrei Nikolaevich, 6, 24, 177
see also Kolmogorov's equation divergent Fourier sefies, 152
life story, 151-15' 172-174
Monte Carlo Sinulation, 153
papers, 170-171
probability theory, 170-171
transition probabilities, 171
work of, 170-171, 172
Kolmogorov School, 174
Kolmogorova, Maria, 173
Kolmogorov's equation see also Kolmogorov, Andrei

Nikolaevich
concepts, 159-162
critical assumption, 171
detailed example, 162-167
differential equation, 160-161, 167-168, 170
ruin probability mathematics, 159 , 161-162, 167-170
verification that equation satisfied, 167-170
Kotlikoff, Laurence, 94

## L

Laplace, Pierre-Simon, 159
law of mortality. See Gompertz law of mortality
League of Nations, 96
legacy, 5-6, 132-134, 140
Lehman Brothers, 124
Levant Company $\boldsymbol{7}_{1}$
Liber Abaci FFihonacci), 17-20, 23, 24, $26,07,29,176$
life annulites. See annuities
Life LT tension Institute, 96
1.: e insurance, 5-6, 51
see also Huebner, Solomon
amount of, 132
buy-term-and-invest-the-difference (BTID) philosophy, 143-144
death benefit, 134-136
human life value, 128-132
legacy, creation of, 132-134
purpose of, 128-132
term $v$ s. whole life, 143-144
valuation of, 134-137
life insurance equation
application of equation, 139-140
different ages, examples, 137-138
the formula, 134-137
life settlements, 142-143
mortality rates, 139-140
permanent life insurance policy, 139
secondary markets, 142-143
life insurance industry, 125-128
see also Huebner, Solomon
life settlements, 142-143
life table, 34-35, 35t, 54
lifecycle consumption smoothing, 82, 94
lifetime horizon, 33-34
lifetime ruin probability with balanced portfolio, $163 t$
logarithm
common logarithms, 11, 180
of infinity, 17
invention of, 23
natural logarithm. See natural logarithm
London Stock Exchange (LSE), 52
long working career, 116
longevity risk, 4, 77
see also Gompertz law of mortality
longevity risk aversion, 86-87, 88-90, 92, 93
longevity uncertainty, 85
lump sums, $v$ s. pensions, 56-59, 71
Lundberg, Filip, 159

## M

Makeham, William, 51-54
market prices, 73
Markov, Andrei, 171
Markov procese 171
Markowitz, 1Fary, 150, 177
Marshall, Itred, 95, 110
Mathematical Annals, 170
mathematical equations. See equations
"Mathematical Investigations in the Theory of Value and Price" (Fisher), 94
maximum theoretical length of life, 62
Mayakovsky, Vladimir, 153
median remaining lifetime, 41, 44-45
Mehra, Rajnish, 104
Menger, Carl, 95

Merton, Robert, 116, 119, 123, 177
"method of trips," 20
Milevsky, Maya A.T., 199-200
modal value of life, 39
modern portfolio theory, 150
Modigliani, Franco, 146
Monte Carlo Simulation, 153, 170
Montefiore, Moses, 52
Morgan, William, 141
mortality patterns as function of age, 54
mortality rate, $35-36,47-48,47 f, 48 f$, 51, 131f
see also Gompertz law of mortality; natural logarihin
mortality table, $34-35,35 t$
see also life table
mutual funds, 177

## N

Na,ier, John, 23
"Narrow Banking," 94
natural exponent, 39, 46, 179
natural exponent function, 179
natural logarithm, 11-12, 23, 36, 179-183
see also logarithm
consumption rate, 84
$e=2.71828,180-181,182-183$
instantaneous force of mortality (IFM), 160
and instantaneous force of mortality (IFM), 169
of mortality rates, 36-37, 39-40, 43, $45-46,47-48,47 f, 48 f, 84-85$
natural logarithm function, 179
negative senescence, 50
nest egg. See principal
Newton, Isaac, 36, 51, 60, 74, 76
Nobel Prize, 123, 177
nominal dollars, 68
nominal interest, 13

## INDEX

nominal interest rate, 80
normative economics, 86
The Number (Eisenberg), 22

## 0

odds, 101-102
optimal retirement portfolio, 120
option pricing equation, 123, 177

## P

Pareto, Vilfredo, 95
patience, 4-5
see also subjective discount rate
pension annuity factor, 61
pensionization, 164, 167
pensions, 4
see also Halley's equation
age and interest, 69-73
choices, $58 t$
Defined Benefit (DB) plan, 56-58
Defined Contribution (DC) plan, 32, 57-58
inflation adjustments, 71
interest rate, effect of, 65-66
vs. lump sum, 56-59, 71
real $v s$. nominal dollars, 68
in 17th century England, 53-50
surviving spouse, 71-7?
valuation of pension annuity, 62-66, 69-73
perihelion, 76
Philosophical Transactions (Royal Society), 55
Pisano, Leonardo, 18
see also Fibonacci, Leonardo
Planck, Max, 119
poem, 199-200
positive economics, 86
preferences, 84
Prescott, Edward, 104
present value, 20, 21, 23-24, 63
present value analysis, 8

Price, Richard, 141
principal
Fibonacci's equation, 10-17
and inflation, 79-80
living off your principal, 10-11
present value equal to initial nest egg, 21
solving for principal required, 21
spending, as you age. See Fisher's
equation
Principles (Newton), 60
probability
of dying, 37,49
see also Gompertz law of mortality
Gambler's Ruin problems, 158-159
of lifetime ruin, 160, 161
lifetime ruin probability with balanced porifolio, $163 t$
ruin probability mathematics, 159
of shortfall 106-107
of survival, 41-43, 42t, 45-46, $49-50,62,63,84-85,160$
ore also Gompertz law of mortality
transition probabilities, 171
-p :obability theory, 170-171
see also Kolmogorov, Andrei Nikolaevich
Prohibition, 96
protected equity investments, 120
Pushkin, A.S., 153

## R

rabbit series, 27-29
rational consumers, 78
real dollars, 68
real interest, 13-14
real interest rate, $21,80,84,85$
reality check, 2-6
recency effect, 103-104
reduced standard of living, 10
retirement expenditures by aging population, $83,83 f$

## INDEX

retirement income planning retirement goal, 22
retirement savings plan, 56
see also pensions
sustainability of retirement plan, 6, 154-158, 162-167, 170
retirement ruin, 172
retirement sustainability analysis, 107
risk
attitudes toward, 87
financial risk, 5
longevity risk, 4, 77
see also Gompertz law of mortality
longevity risk aversion, 86-87, 88-90, 92, 93
risk aversion, 85, 113
risk aversion, 85, 113
risky stocks, 107-108
see also Samuelson, Paul
Roman numerals, 26
Roosevelt, Franklin D., 6, 120
Rothschild, Nathan, 52
roulette wheel, 159
Royal Society, 54, 55, 60, 73
ruin probability mathematics, 159 , 161-162, 163t, 16 -170
see also Kolmogorpresequation
Russia, 151-152
see also Kolmogorov, Andrei Nikcłacylch
Rye House Piot, 55-56

## S

safest possible investment, 112-113
Samuelson, Paul, 5, 94, 101-102, 157-158
see also Samuelson's equation
impact, 120-124
life story, 120-124
Nobel Prize, 123
on risky investments, 107-108
time, relevance of, 107-108
on time, and risk attitudes, 118-119
Samuelson, William, 119
Samuelson's equation
see also Samuelson, Paul
application of, 114-119
case studies and examples, 114-119
explained, 111-114
tips and insights, 119-120
variables, 111-113
Scholes, Myron, 123, 177
Schumpeter, Joseph, 95
scientific approach, 33-34
second derivative, 162,169
secondary markets, $142,-143$
Sense and Sensibility (Austen), 67
sequence-of-retwn (SoR) effect, 107
sex, and motality, 44-45
Sharre, William, 177
Shiller, Robert, 104
Siu'vet, Trevor, 142
Siegel, Jeremy, 104
Sigler, Laurence, 17
Skull and Bones Society, 94
Smith, Adam, 122
Social Security, 6
Spanish Flu, 130
spending rate
see also Fisher's equation
constant spending rate, 92
Fibonacci's equation, 9-17
retirement expenditures by aging population, $83,83 f$
sustainable, 81-83
Standard \& Poor's 500 index, 105
standard deviation, 161
Stiglitz, Joseph, 121
stochastic differential equation (SDE), 172
stock market
see also Samuelson's equation
crash of 1929, 98-100, 149

## INDEX

crash of 1987, 103
in long run, 104-108
mid-to-late 1990s, 102-104
probability of shortfall, 106-107
stocks
see also Samuelson's equation
expected growth rate, 112
optimal retirement portfolio, 120
risky stocks, 107-108
volatility of stocks, 112, 119-120
Stocks for the Long Run (Siegel), 104
Stone, Mildred F., 127, 149
structured equity products, 120
subjective discount rate, $84,85-86$, 88-89, 92
subjective time preference, 4
Sunny Sol. See Huebner, Solomon
survival probability. See Gompertz law of mortality; probability
sustainability, 6, 154-158, 162-167, 170
see also Kolmogorov's equation

## T

Taft, William H., 97, 120
tastes, 84
The Teacher Who Changed an Inaustry (Huebner), 149
term life insurance, 143-144
Thaler, Richard, 95
The Theory of Intere.t: As Determined by Impationce to Spend Income
and Opportunity to Invest It (Fisher), 87, 95
time diversification, 107
time invariance, 107-108, 109
time on your side, question of, 102-109
Tooke, Mary, 74
Tosca, Floria, 95
transition probabilities, 171
Truman, Harry S., 157
Twain, Mark, 76

## U

utility value, 86

## V

"Value of Life Annuities in Proportion to Redeemable Annuities" (de Wite, 110
volatility, 112, 119-120, 161

## W

Wat:as, Leon, 95
Wharton School, 144, 149
whole life insurance, 143-144
withdrawal rate, 9-17

## Y

Yaari, Menahem, 87, 146







