

# Preface

In 1992, Heath, Jarrow, and Morton (1992) (HJM) developed a general framework to model the dynamics of the entire forward rate curve in an interest rate market. The associated valuation approach is based on two main assumptions: the first one postulates that it is not possible to gain riskless profit (no-arbitrage condition), and the second one assumes the completeness of the financial market. The HJM model, or strictly speaking the HJM framework, is a general model environment and incorporates many previously developed models like the model of Ho and Lee (1986), Vasicek (1977) and Hull and White (1990). The general setting mainly suffers from two disadvantages: first of all the difficulty to apply the model in market practice and second the extensive computational complexity caused by the high-dimensional stochastic process of the underlying. The first disadvantage was improved by the development of the LIBOR market model introduced by Brace, Gatarek, and Musiela (1997), Jamshidian (1997) and Miltersen and Sandmann (1997), which combines the general risk-neutral yield curve model with market standards. The second disadvantage can be improved by restricting the general HJM model to a subset of models with a specific parametrization of the volatility function. The resulting system of stochastic differential equations (SDE) describing the yield curve dynamics breaks down from a high-dimensional process into a low-dimensional structure of Markovian processes. This approach was developed by Cheyette (1994).

In practice, the Cheyette models usually incorporate several factors to achieve sufficient flexibility to represent the market state. The structure of the models supports a canonical construction of multifactor models as extensions of the one-factor model. The model dynamics consider all factors and might become a high-dimensional SDE as each factor captures at least one dimension.

The purpose of this work is to show the application of the class of Cheyette models in practice. Therefore, we focus on the necessary topics, namely, calibration, valuation and sensitivity analysis. Since we work in a Gaussian HJM framework, some analytical formulas can be derived and various numerical methods are applicable. In this book, we present the methods in detail and highlight the improvements due to the special structure of Cheyette models. Thereby the main focus is not on the latest implementation details, but we show the impact of the chosen interest rate model.

The class of Cheyette models is part of the general Gaussian HJM framework, which is characterized by a specific parametrization of the volatility incorporating a wide range of functions. Thus, the corresponding interest rate models become very flexible and could capture almost all changes to the term structure of interest. This positive effect is even intensified by including several stochastic factors, which can be done easily in this setup. Hence, we deal with Cheyette models throughout this book, which are Gaussian HJM models with time-dependent volatility.

The price of interest rate derivatives can be expressed as the expected value of the terminal payoff under given model dynamics. Therefore, the computation comes up as a multidimensional integral, which might be difficult to solve. Consequently, academics and practitioners have become interested in methods to handle these model dynamics and compute accurate prices of interest rate derivatives in reasonable time. Thereby, one can exploit the special structure of the class of Cheyette models and achieve efficient valuation techniques as we see in this work.

In addition to pricing derivatives, practitioners always highlight the hedging of financial positions and, thus, focus on risk sensitivities, the so-called Greeks. Since we can derive analytical pricing formulas for bonds and caplets, we are able to derive analytical Greek formulas for Model- and Market-Greeks in these cases as well. Furthermore, we demonstrate how to compute Model- and Market-Greeks for exotic interest rate derivatives numerically.

The book is divided into ten chapters and is organized as follows:

After a review of existing literature in Chap. 1, Chap. 2 introduces to the class of Cheyette models. First, we present the general HJM framework and imposing a structure on the volatility function directly leads to the class of Cheyette models. The setup covers well-known models like the Ho-Lee, the Hull-White or Vasicek model. We explain the representation of these models in the Cheyette framework, and finally, we distinguish the approach from alternative specifications of the HJM framework.

In Chap. 3, we present analytical pricing formulas for bonds and caplets/floorlets, which are valid in the whole class of Cheyette models. The bond pricing formula has been published by Cheyette (1994) in the case of one-factor models only. Furthermore, we derive explicit pricing formulas for caplets and floorlets based on some general results of Musiela and Rutkowski (2005). Applying their results to the class of Cheyette models leads to these pricing formulas.

One of the most important and challenging parts of pricing interest rate derivatives is the model calibration to a given market state. In Chap. 4, we formulate the calibration problem and present some methodologies to solve it in the class of Cheyette models. The theoretical aspects are illustrated by the calibration of a multifactor model, which serves as a reference throughout the book.

Chapter 5 treats the valuation of interest rate derivatives via Monte Carlo simulation. Since we derive the distribution of the state variables, we obtain very fast and accurate algorithms to value plain-vanilla and exotic derivatives. Implementing a quasi-Monte Carlo simulation accelerates the evaluation further. Monte Carlo simulation is a very robust method, and we apply it to value all kinds of plain vanilla and exotic derivatives. Thus, we use Monte Carlo simulation as a stand-alone

method as well as a reference method to different valuation techniques based on characteristic functions and PDEs. Furthermore, we pick up Monte Carlo simulation again in Chap. 9 for the computation of risk sensitivities.

In Chap. 6, we develop characteristic functions of Cheyette models as the solution to a system of certain complex-valued ordinary differential equations. Due to the structure of Cheyette models, these Riccati equations can be solved explicitly, and we end up with a fast and accurate pricing methodology for bonds and caplets.

In Chap. 7, we derive the PDE to value interest rate derivatives. In general, the computation via PDE is difficult, because the valuation PDE might become high dimensional. So far, the application was limited to two-dimensional problems. We use the sparse grid technique based on Finite Differences to solve the terminal value problem numerically. The methodology was applied successfully to high-dimensional problems by Reisinger (2004) and Bungartz and Griebel (2004) in the case of stock options. We transfer the technique to interest rate derivatives and make use of some modifications developed by Chiarella and Kang (2012).

In Chap. 8, we summarize the applicability, numerical tractability and accuracy of the previously presented pricing methodologies. Therefore, we analyze and compare the approaches for plain-vanilla and exotic interest rate derivatives.

Chapter 9 deals with risk sensitivities, and we derive analytical formulas for bonds and caplets. In the case where no analytical formulas are applicable, we show how to compute the required Greeks numerically using Monte Carlo simulation.

Please note that the book is a revised version of my dissertation at Frankfurt School of Finance & Management in the Centre for Practical Quantitative Finance with the title ‘Valuation, Calibration and Sensitivity Analysis of Interest Rate Derivatives in a Multifactor HJM Model with Time Dependent Volatility’.

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