

# Introduction

The number of alternative investments is overwhelming. Thousands of stocks, thousands of bonds, and many other alternatives are worthy of consideration. The purpose of this book is to simplify the investor's choices by treating the countably infinite number of stocks, bonds, and other individual assets as components of portfolios. Portfolios are the objects of choice. The individual assets that go into a portfolio are inputs, but they are not the objects of choice on which an investor should focus. The investor should focus on the best possible portfolio that can be created.

Portfolio theory is not as revolutionary as it might seem. A portfolio is simply a list of assets. But managing a portfolio requires skills.

## 1.1 THE PORTFOLIO MANAGEMENT PROCESS

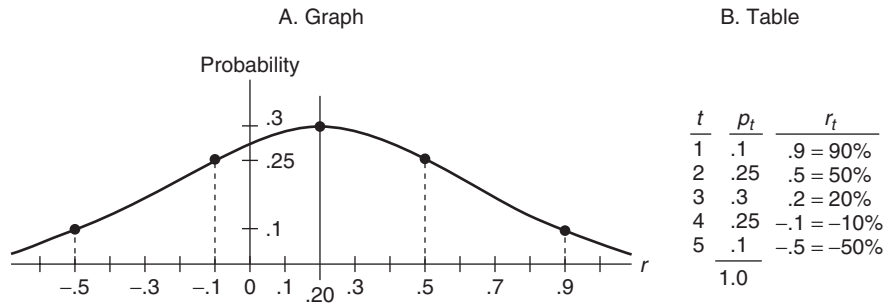
The portfolio management process is executed in steps.

- Step 1. **Security analysis** focuses on the probability distributions of returns from the various investment candidates (such as individual stocks and bonds).
- Step 2. **Portfolio analysis** is the phase of portfolio management that delineates the optimum portfolio possibilities that can be constructed from the available investment opportunities.
- Step 3. **Portfolio selection** deals with selecting the single best portfolio from the menu of desirable portfolios.

These three phases are discussed briefly below.

## 1.2 THE SECURITY ANALYST'S JOB

Part of the security analyst's job is to forecast. The security analyst need not forecast a security's returns for many periods into the future. The forecaster only needs to forecast security returns that are plausible for one period into the future. The length of this one-period forecasting horizon can vary within wide limits. It should not be a short-run period (such as an hour or a day), because portfolio analysis is not designed to analyze speculative trading. The forecasting horizon cannot be very long either, because it is not realistic to assume the security analyst is prescient. Between one month and several years, the portfolio manager can select any planning horizon that fits comfortably within the portfolio owner's holding period (investment horizon).



**FIGURE 1.1** Tom's Subjective Probability Distribution of Returns

The security analyst's forecast should be in terms of the holding period rate of return, denoted  $r_1$ . For instance, for a share of common or preferred stock,  $r_1$  is computed as follows.

$$\left( \text{One-period rate of return} \right) = \frac{\left( \text{Price change during the holding period} \right) + \left( \text{Cash dividends paid during the holding period, if any} \right)}{\left( \text{Purchase price at the beginning of the holding period} \right)}$$

$$\text{or } r_1 = \frac{(P_1 - P_0) + d_1}{P_0} \quad (1.1)$$

where  $P_0$  denotes the price of a share of stock at the beginning of the holding period,  $P_1$  represents the price at the end of the holding period, and  $d_1$  stands for any cash dividend that might have been paid during the holding period (typically one month or one year).<sup>1</sup>

The security analyst should construct a probability distribution of returns for each individual security that is an investment candidate. The needed rates of return may be compiled from historical data if the candidate security already exists (that is, is not an initial public offering). The historically derived probability distribution of returns may then need to be adjusted subjectively to reflect anticipated factors that were not present historically. Figure 1.1 provides an example of a probability distribution of the rates of return for Coca-Cola's common stock that was constructed by a security analyst named Tom. This probability distribution is a finite probability distribution because the outcomes (rates of return) are assumed to be discrete occurrences.

The security analyst must also estimate correlation coefficients (or covariances) between all securities under consideration. Security analysis is discussed more extensively in Chapters 2 and 8. The expected return, variance, and covariance statistics are the input statistics used to create optimal portfolios.

### 1.3 PORTFOLIO ANALYSIS

Portfolio analysis is a mathematical algorithm created by the Nobel laureate Harry Markowitz during the 1950s.<sup>2</sup> Markowitz portfolio analysis requires the following statistical inputs.

- The expected rate of return,  $E(r)$ , for each investment candidate (that is, every stock, every bond, etc.).
- The standard deviation of returns,  $\sigma$ , for each investment candidate.
- The correlation coefficients,  $\rho$ , between all pairs of investment candidates.

Markowitz portfolio analysis takes the statistical inputs listed above and analyzes them simultaneously to determine a series of plausible investment portfolios. The solutions explain which investment candidates are selected and rejected in creating a list of optimal portfolios that can achieve some expected rate of return. Each Markowitz portfolio analysis solution also gives exact portfolio weightings for the investment candidates in that solution.

### 1.3.1 Basic Assumptions

Portfolio theory is based on four behavioral assumptions.

1. All investors visualize each investment opportunity (for instance, each stock or bond) as being represented by a probability distribution of returns that is measured over the same planning horizon (holding period).
2. Investors' risk estimates are proportional to the variability of the returns (as measured by the standard deviation, or equivalently, the variance of returns).
3. Investors are willing to base their decisions on only the expected return and risk statistics. That is, investors' utility of returns function,  $U(r)$ , is solely a function of variability of return ( $\sigma$ ) and expected return [ $E(r)$ ]. Symbolically,  $U(r) = f[\sigma, E(r)]$ . Stated differently, whatever happiness an investor gets from an investment can be completely explained by  $\sigma$  and  $E(r)$ .
4. For any given level of risk, investors prefer higher returns to lower returns. Symbolically,  $\partial U(r) / \partial E(r) > 0$ . Conversely, for any given level of rate of return, investors prefer less risk over more risk. Symbolically,  $\partial U(r) / \partial \sigma < 0$ . In other words, all investors are risk-averse rate of return maximizers.

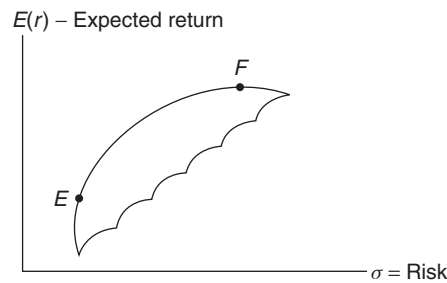
### 1.3.2 Reconsidering the Assumptions

The four behavioral assumptions just listed are logical and realistic and are maintained throughout portfolio theory. Considering the four assumptions implies the most desirable investments have:

- The minimum expected risk at any given expected rate of return. Or, conversely,
- The maximum expected rate of return at any given level of expected risk.

Investors described by the preceding assumptions will prefer Markowitz efficient assets. Such assets are almost always portfolios rather than individual assets. The Markowitz efficient assets are called *efficient portfolios*, whether they contain one or many assets.

If all investors behave as described by the four assumptions, portfolio analysis can logically (mathematically) delineate the set of efficient portfolios. The set of efficient portfolios is called the *efficient frontier* and is illustrated in Figure 1.2. The efficient portfolios along the curve between points E and F have the maximum rate of return at each level of risk. The efficient frontier is the menu from which the investor should make his or her selection.



**FIGURE 1.2** The Efficient Frontier

Before proceeding to the third step of the portfolio management process, portfolio selection, let us pause to reconsider the four assumptions listed previously. Portfolio theory is admittedly based on some simplifying assumptions that are not entirely realistic. This may raise questions in some people's minds. Therefore, we will examine the validity of the four assumptions underlying portfolio theory.

The first assumption about probability distributions of either terminal wealth or rates of return may be violated in several respects. First, many investors simply do not forecast assets' prices or the rate of return from an investment. Second, investors are frequently heard discussing the "growth potential of a stock," "a glamor stock," or the "quality of management" while ignoring the investment's terminal wealth or rates of return. Third, investors often base their decisions on estimates of the most likely outcome rather than considering a probability distribution that includes both the best and worst outcomes.

These seeming disparities with assumption 1 are not serious. If investors are interested in a security's glamor or growth, it is probably because they (consciously or subconsciously) believe that these factors affect the asset's rate of return and market value. And even if investors cannot define rate of return, they may still try to maximize it merely by trying to maximize their net worth: Maximizing these two objectives can be shown to be mathematically equivalent. Furthermore, forecasting future probability distributions need not be highly explicit. "Most likely" estimates are prepared either explicitly or implicitly from a subjective probability distribution that includes both good and bad outcomes.

The risk definition given in assumption 2 does not conform to the risk measures compiled by some popular financial services. The quality ratings published by Standard & Poor's are standardized symbols like AAA, AA, A, BBB, BB, B, CCC, CC, and C. Studies suggest that these symbols address the probability of default. Firms' probability of default is positively correlated with their variability of return. Therefore, assumption 2 is valid.<sup>3</sup>

As pointed out, investors sometimes discuss concepts such as the growth potential and/or glamor of a security. This may seem to indicate that the third assumption is an oversimplification. However, if these factors affect the expected value and/or variability of a security's rate of return, the third assumption is not violated either.

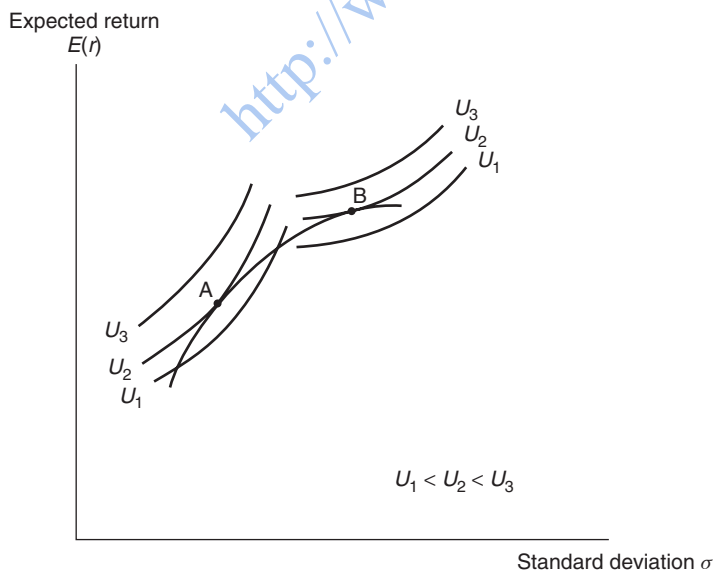
The fourth assumption may also seem inadequate. Psychologists and other behavioralists have pointed out to economists that business people infrequently maximize profits or minimize costs. The psychologists explain that people usually

strive only to do a satisfactory or sufficient job. Rarely do they work to attain the maximum or minimum, whichever may be appropriate. However, if some highly competitive business managers attain near optimization of their objective and other business managers compete with these leaders, then this assumption also turns out to be realistic.

All the assumptions underlying portfolio analysis have been shown to be simplistic, and in some cases overly simplistic. Although it would be nice if none of the assumptions underlying the analysis were ever violated, this is not necessary to establish the value of the theory. If the analysis rationalizes complex behavior (such as diversification), or if the analysis yields worthwhile predictions (such as risk aversion), then it can be valuable in spite of its simplified assumptions. Furthermore, if the assumptions are only slight simplifications, as are the four mentioned previously, they are no cause for alarm. People need only behave *as if* they were described by the assumptions for a theory to be valid.<sup>4</sup>

## 1.4 PORTFOLIO SELECTION

The final phase of the portfolio management process is to select the one best portfolio from the efficient frontier illustrated in Figure 1.2. The utility of returns function, which aligns with the four basic assumptions previously listed, is very helpful in selecting an optimal portfolio. Utility of return functions can be formulated into indifference curves in  $[\sigma, E(r)]$  space. Two different families of indifference curves that were created from similar but different utility of return functions are illustrated in Figure 1.3 to represent the preferences of two different investors. Figure 1.3 shows investor B achieves his maximum attainable happiness from investing in a riskier



**FIGURE 1.3** Different Optimal Portfolios for Different Investors

efficient portfolio than investor A's. In other words, investor A is more risk-averse than investor B.

Portfolio selection is made more difficult because security prices change as more recent information continually becomes available. And as cash dividends are paid, the expected return and risk of a selected portfolio can migrate. When this happens, the portfolio must be revised to maintain its superiority over alternative investments. Thus, portfolio selection leads, in turn, to additional security analysis and portfolio analysis work. Portfolio management is a never-ending process.

## **1.5 THE MATHEMATICS IS SEGREGATED**

What follows can be mathematical. However, the reader who is uninitiated in mathematics can master the material. All that is needed is a remembrance of freshman college algebra, one course in statistics, and persistent interest. Most of the chapters are written at the simplest level that a fair coverage of the model allows. The basic material is presented completely in terms of elementary finite probability theory and algebra supplemented with graphs, explanations, examples, and references to more complete explanations.

Differential calculus and matrix algebra are used in a few chapters (6, 7, 13, 15) and in some of the appendixes. The reader is hereby forewarned and may avoid this material. The book is written so its continuity will not be disturbed by skipping these chapters and appendixes. None of the vocabulary or basic concepts necessary for an acquaintance with the subject is found in the few mathematical chapters and end-of-chapter appendixes. Most of the appendixes contain mathematical solution techniques for large problems, proofs, derivations, and other material of interest only to the so-called rocket scientists.

## **1.6 TOPICS TO BE DISCUSSED**

This book addresses the following aspects of portfolio analysis:

1. *Probability foundations*: This monograph explains probabilistic tools with which risk may be analyzed. See Chapter 2.
2. *Utility analysis*: Chapter 4 focuses on the investor's personal objective, stated in terms of his or her preferences for risk and return.
3. *Mean-variance portfolio analysis*: Portfolio analysis delineates the optimum portfolio possibilities that can be constructed from the available investment opportunities, assuming asset returns are normally distributed. See Chapters 5, 6, and 7.
4. *Non-mean-variance portfolio analysis*: This approach to portfolio analysis delineates the optimum portfolio possibilities when asset returns are not normally distributed. See Chapter 10.
5. *Asset pricing models*: This extension of portfolio analysis investigates models that provide suggestions about the appropriate risk-return trade-off. See Chapters 12 – 16.
6. *Implementation of portfolio theory*: This phase of portfolio analysis is concerned with the construction of the set of optimal portfolios from which an investor can

select the best portfolio based on his or her personal objectives. See Chapters 11 and 19.

7. *Periodic performance evaluations*: The investment performances of invested portfolios should be analyzed to ascertain what is right and what is going wrong. See Chapter 18,

Portfolio analysis deals with only one time period. It assumes that the investor has a given amount of investable wealth and would like to identify, *ex ante* (before the fact), the optimal portfolio to purchase for the next time period. The selected portfolio may not turn out to be optimal *ex post* (after the fact), meaning that it may not turn out to have been the portfolio with the highest realized rate of return. Because the future cannot be forecast perfectly, all portfolio returns can be viewed as random variables. Portfolio theory recognizes this and suggests that the portfolio manager identify the best portfolio by evaluating all portfolios in terms of their risk and expected returns and then choosing the one that best fits his or her preferences.

Although this book uses some mathematical and statistical explanations, the reader who is only slightly initiated in mathematics and statistics can master the basic material. All that is needed is a remembrance of freshman college algebra and calculus, one course in classical statistics, and patience. The material is presented at the simplest level that a fair coverage of the models will allow and is presented in terms of elementary mathematics and statistics supplemented with graphs.

## **APPENDIX: VARIOUS RATES OF RETURN**

The one-period rate of return may be defined in several different, but similar, ways. This appendix considers some alternatives.

### **A1.1 Calculating the Holding Period Return**

If an investor pays a price ( $P_0$ ) for a stock at the beginning of some period (say, a year) and sells the stock at a price ( $P_1$ ) at the end of the period after receiving dividends ( $d_1$ ) during the period, the rate of return for that holding period,  $r_1$ , is the discount rate that equates the present value of all cash flows to the cost of investment. Symbolically,

$$P_0 = \frac{(P_1 + d_1)}{1 + r_1} \quad \text{or} \quad r_1 = \frac{[(P_1 - P_0) + d_1]}{P_0} \quad (\text{A1.1})$$

Thus, if \$100 is invested for one year and returns the principal plus capital gains of  $(P_1 - P_0) = \$7$ , plus \$8 of cash dividends, the rate of return is calculated using equation (A1.1) as follows.

$$r_1 = \frac{[(P_1 - P_0) + d_1]}{P_0}$$

$$r_1 = [(\$107 - \$100) + \$8] / \$100 = 15\% \quad \text{annual rate of return.}$$

The rate of return defined by equation (A1.1) is frequently called the investor's holding period return (HPR).

Equation (A1.1) is defined in terms of the income sources from a common or preferred stock investment, because this analysis is frequently concerned with portfolios of stocks. However, the rates of return from other forms of investment are easily defined, and this analysis is general enough so that all kinds of assets may be considered. For example, the rate of return from a coupon-paying bond is

$$r_1 = \frac{[(P_1 - P_0) + c]}{P_0} \quad (\text{A1.2})$$

where  $c$  denotes the coupon interest paid during the holding period.

The rate of return from a real estate investment can be defined as

$$r_1 = (V_1 - V_0) / V_0 \quad (\text{A1.3})$$

where  $V_1$  is the end-of-period value for a real estate holding and  $V_0$  is the beginning-of-period value. If the investor receives rental income from the real estate, then equation (A1.3a) is appropriate.

$$r_1 = (V_1 - V_0 + \text{Rental Income}) / V_0 \quad (\text{A1.3a})$$

### A1.2 After-Tax Returns

This analysis can be conducted to fit the needs of an investor in a given tax situation. That is, equation (A1.1) may be adapted to treat tax differentials between the income from cash dividends and capital gains. This would require restating equation (A1.1) in the form

$$r_1 = \frac{[(P_1 - P_0)(1 - \tau_G) + d_1(1 - \tau_o)]}{P_0} \quad (\text{A1.4})$$

where  $\tau_G$  is the relevant capital gains tax rate and  $\tau_o$  is the relevant ordinary income tax rate that is appropriate for the particular investor's income. In addition to the effect of taxes, brokerage commissions and other transactions costs can be included in the computations. Equation (A1.4a) defines a stock's one-period return after commissions and taxes:

$$r_1 = \frac{[(P_1 - P_0 - \text{Commissions})(1 - \tau_G) + d_1(1 - \tau_o)]}{P_0} \quad (\text{A1.4a})$$

Unless otherwise stated, the existence of taxes and transaction costs such as commissions will be ignored to allow us to proceed more simply and rapidly.

### A1.3 Discrete and Continuously Compounded Returns

Equations (A1.1), (A1.2), and (A1.3) define a holding period rate of return that is compounded once per time period. If the rate of return for a one-year holding period is 12 percent, a \$1 investment will grow to  $\$1(1 + r) = \$1(1 + 0.12) = \$1.12$

after one year. If the rate is compounded semiannually (two times a year), at what rate of return will the \$1 investment become \$1.12? In other words, what is the semiannually compounded rate of return? The answer is

$$\begin{aligned} \$1 (1 + 0.12) &= \$1 \left(1 + \frac{r_{\text{semi}}}{2}\right)^2 \\ \Rightarrow r_{\text{semi}} &= 2 \times [(1 + 0.12)^{1/2} - 1] = 0.1166 \end{aligned}$$

That is, if half of the semiannually compounded return of 11.66 percent is compounded twice a year, it will be the same as the holding period return of 12 percent that is compounded only once per year. The monthly compounded rate of return is

$$\begin{aligned} \$1 (1 + 0.12) &= \$1 \left(1 + \frac{r_{\text{monthly}}}{12}\right)^{12} \\ \Rightarrow r_{\text{monthly}} &= 12 \times [(1 + 0.12)^{1/12} - 1] = 0.1139 \end{aligned}$$

If a twelfth of the monthly compounded return of 11.39 percent is compounded for 12 months, it will equal the holding period return of 12 percent. In general, when the holding period return is  $r_t$  and the number of compounding is  $m$  times per period, the  $m$ -compounded rate of return is computed from

$$r_{m\text{-compounded}} = m \times \left[ (1 + r_t)^{1/m} - 1 \right] \quad (\text{A1.5})$$

Equation (A1.5) can be rewritten as

$$\left(1 + \frac{r_{m\text{-compounded}}}{m}\right)^m = 1 + r_t \quad (\text{A1.5a})$$

The continuously compounded return (the frequency of compounding is infinity per period),  $\dot{r}_t$ , is computed from the following equation:

$$\lim_{m \rightarrow \infty} \left(1 + \frac{\dot{r}_t}{m}\right)^m \equiv e^{\dot{r}_t} = 1 + r_t \quad (\text{A1.6})$$

Thus, the continuously compounded return is

$$\dot{r}_t = \ln(1 + r_t) \quad (\text{A1.7})$$

where  $\ln$  denotes the natural (or Napierian) logarithm. Hereafter,  $\dot{r}_t$  will be referred to as the continuously compounded rate of return or, more concisely, the continuous return, and  $r_t$  will be referred to as the holding period return, the noncompounded rate of return or, simply, the return. The continuously compounded rate of return is always less than the holding period return; that is,  $\dot{r}_t < r_t$ . If  $r_t$  is small, these two returns will be close. Thus, if returns are measured over a short period of time, such as daily, the continuously compounded returns and the noncompounded returns would be quite similar.

The continuously compounded rate of return from a stock for a given period, assuming no cash dividend payments, can also be computed as the difference between two natural log prices at the end and beginning of the period. That is,

$$\dot{r}_t = \ln(P_t/P_{t-1}) = \ln P_t - \ln P_{t-1} \quad (\text{A1.8})$$

## NOTES

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1. Similar but different definitions for the rate of return may be found in the Appendix to this chapter.
2. For his collected works see Harry M. Markowitz, Editor, *Harry Markowitz Selected Works*, 2010, World Scientific Publishing Company, Hackensack, New Jersey, ISBN-13 978-981-283-363-1.
3. Frank J. Fabozzi, "The Association between Common Stock Systematic Risk and Common Stock Rankings," *Review of Business and Economic Research* 12, no. 3 (Spring 1977): 66–77.
4. Milton Friedman, "The Methodology of Positive Economics," *Essays in Positive Economics* (Chicago: University of Chicago Press, 1953), 3–43.

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